The Legendre polynomials

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$$

can be normalized on [-1,1] to

$$\mathbb{P}_n = \sqrt{\frac{2n+1}{2}} \frac{1}{2^n n!} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$$

Note that the Legendre polynomials are orthogonal.

$$\int_{-2}^{2} P_n(2, x)\, dx = 6 \qquad \int_{-2}^{2} P_n(2, x)\, P_n(3, x)\, dx = 0$$

When calculating the Legendre coefficients to use with Legendre polynomials to approximate functions, it is useful to calculate a weighting function that normalizes the polynomials.

$$n := 0..3 \qquad m := 0..3 \qquad \int_{-1}^{1} P_n(x)\, P_m(n, x)\, dx = \frac{2}{2n+1}\delta(n, m)$$

$$c_{n,m} := \int_{-1}^{1} P_n(n, x)\, P_m(m, x)\, dx$$

$$c = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.667 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0.286 \end{pmatrix}$$

Approximate f(x) using Legendre polynomials.

$$f(x) = \sum_n \left(c_n P_n(x)\right) \Rightarrow \left(f, P_m\right) = \sum_n \left[c_n\left(P_n, P_m\right)\right] = c_m \frac{2}{2m+1} \Rightarrow c_n = \frac{2n+1}{2}\left(P_n, f\right)$$

$$f(x) := \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \qquad n := 0..9 \qquad c_n := \frac{2n+1}{2}\int_{-1}^{1} f(x)\, P_n(n, x)\, dx \qquad L(x) := \sum_n \left(c_n P_n(n, x)\right)$$