0.1 The Eigenvalue Problem

$$\left.\begin{aligned}
&\frac{d^2\Psi(x^*)}{dx^{*\,2}} - l^2\Psi(x^*) = 0, \quad \Psi(0) = 0\ , \Sigma(1) = 0,\\
&\frac{d^2\Sigma(x^*)}{dx^{*\,2}} - l^2\Sigma(x^*) = 0,\ \Psi(S_0^*) = \Sigma(S_0^*) + \Phi\, N_0,\\
&\varphi\,(1-R_1)\,\sigma\,\Phi = R\ \left.\frac{\partial\Sigma}{\partial x^*}\right|_{x^*=S_0^*} - \left.\frac{\partial\Psi}{\partial x^*}\right|_{x^*=S_0^*}.
\end{aligned}\right\} \tag{1}$$

The solutions of (1) for the pressure eigenfunctions subject to the pressure continiuty conditions (1) are

$$\left.\begin{aligned}
&\Psi(x^*) = 2\,C_3\,\sinh(\,l\,x^*),\\
&\Sigma(x^*) = 2\,C_4\,\frac{\sinh(\,l\,(x^*-1))}{\cosh(\,l) - \sinh(\,l)},
\end{aligned}\right\} \tag{2}$$

where the constant of integration are

$$C_3 = \frac{1}{2}\frac{\Phi}{l}\left[\frac{\Big\{\varphi\,\sigma\,(1-R_1) + \coth(l\,(S_0^*-1))\,l\,N_0\Big\}}{\sinh(l\,S_0^*)\Big\{R\,\coth(l\,(S_0^*-1)) - \coth(l\,S_0^*)\Big\}}\right],$$

$$C_4 = \frac{1}{2}\frac{\Phi}{l}\left[\frac{\Big\{\varphi\,\sigma\,(1-R_1) + \coth(l\,S_0^*)\,l\,N_0\Big\}\Big\{\cosh(l) - \sinh(l)\Big\}}{\sinh(l\,(S_0^*-1))\Big\{R\,\coth(l\,(S_0^*-1)) - \coth(l\,S_0^*)\Big\}}\right].$$

where

$$N_0 = F_1(S_0^*)\,\frac{C}{k}\,\frac{Pec_i}{R}\left\{(R-1) + \frac{k\,R\,R_3\,(1-R_1)}{C\,Pec_i}\right\}.$$

$$\begin{aligned}
&F(S_0^*) = \frac{\dot m_{liq,vap}}{\dot m_i} = F_1(S_0^*)\,[1 - R\,G_1\,S_0^*]\,, \quad F_1(S_0^*) = \frac{1}{R\,S_0^* + 1 - S_0^*},\\
&G_1 = \frac{R_3\,k(1-R_1)}{C\,Pec_i}.
\end{aligned} \tag{3}$$

The maple code for (1) is in the sheet name Eigenfunpressure.mws.

The eigenvalue problem for the temperature distributions in the both phases,

$$\left.\begin{aligned}
&\left(\frac{d^2}{dx^{*\,2}} - \omega_0 \frac{d}{dx^*} - E_1\,\sigma - l^2\right)\phi_{liq} + \frac{d\Theta^0_{liq}}{dx^*}\,\frac{d\Psi}{dx^*} = 0,\\
&\phi_{liq}(0) = 0, \quad \phi_{liq}(S_0^*) = -\Phi\left.\frac{d\Theta^0_{liq}}{dx^*}\right|_{x^*=S_0^*},\\
&\left(\frac{d^2}{dx^{*\,2}} - \Omega_0 \frac{d}{dx^*} - \frac{E_2\,k\,R_1}{C}\,\sigma - l^2\right)\phi_{vap} + \frac{R\,k}{C}\,\frac{d\Theta^0_{vap}}{dx^*}\,\frac{d\Sigma}{dx^*} = 0,\\
&\phi_{vap}(1) = 0, \quad \phi_{vap}(S_0^*) = -\Phi\left.\frac{d\Theta^0_{vap}}{dx^*}\right|_{x^*=S_0^*},
\end{aligned}\right\} \tag{4}$$

where

$$\left.\begin{aligned}
&\frac{d\Theta^0_{liq}}{dx^*} = N_1\,\exp\left(\omega_0\,x^*\right),\\
&\frac{d\Theta^0_{vap}}{dx^*} = N_2\,\exp\left(\Omega_0\,(x^*-1)\right),
\end{aligned}\right\} \tag{5}$$

with leads to

$$\left.\frac{d\Theta^0_{liq}}{dx^*}\right|_{x^*=S_0^*} = N_3, \quad \text{and} \quad \left.\frac{d\Theta^0_{vap}}{dx^*}\right|_{x^*=S_0^*} = N_4, \tag{6}$$

where

$$\left.\begin{aligned}
&N_1 = \frac{\omega_0}{\exp(\omega_0\,S_0^*) - 1},\ N_2 = \frac{\Omega_0}{\exp(\Omega_0\,(S_0^*-1)) - 1},\\
&N_3 = N_1\,\exp(\omega_0\,S_0^*), \quad N_4 = N_2\,\exp\left(\Omega_0\,(S_0^*-1)\right).
\end{aligned}\right\} \tag{7}$$

and

$$\left.\begin{aligned}
&\omega_0 = \frac{1}{k}\,Pec_i\,F(S_0^*)\,C,\\
&\Omega_0 = Pec_i\,F(S_0^*),
\end{aligned}\right\} \tag{8}$$

The maple code for (4) in particular for the ϕ_{liq} is in the sheet name Eignfunengery liquid.wms and for ϕ_{vap} is in Eignfunengery vapour.mws

0.2 Dispersion Analysis

An equation which represent the relationship between the decay rate σ and the wave number l is known as dispersion equation and can be obtained by using the energy jump across the liquid-vapour interface.

$$\varphi\,\sigma\,H_{liq}\,\Phi = \left\{\Phi\,\frac{d^2\Theta^0_{liq}}{dx^{*\,2}} + \frac{d\phi_{liq}}{dx^*}\right\} + \frac{\Theta_0}{k}\left\{\Phi\,\frac{d^2\Theta^0_{vap}}{dx^{*\,2}} + \frac{d\phi_{vap}}{dx^*}\right\} - H_{liq}\,\frac{d\Psi_1}{dx^*}. \tag{9}$$

From the basic state we know that

$$\left.\begin{aligned} \left.\frac{d^2\Theta^0_{liq}}{dx^{*\,2}}\right|_{x^*=S^*_0} &= N_3\,\omega_0, \\ \left.\frac{d^2\Theta^0_{vap}}{dx^{*\,2}}\right|_{x^*=S^*_0} &= N_4\,\Omega_0. \end{aligned}\right\} \tag{10}$$

Substituting (10) into (9) yields

$$\varphi\,\sigma\,H_{liq}\,\Phi = \left\{\frac{d\phi_{liq}}{dx^*} + \Phi\,N_3\,\omega_0\right\} + \frac{\Theta_0}{k}\left\{\frac{d\phi_{vap}}{dx^*} + \Phi\,N_4\,\Omega_0\right\} - H_{liq}\,\frac{d\Psi_1}{dx^*}. \tag{11}$$

The maple code for (9) are in the sheet with name (dispersion eqn.mws)

$$\frac{\Theta_0}{C} = \{\exp\left[Pec_i\,F(S^*_0)\,(1 - S^*_0)\right] - 1\}\left\{H_{liq} + \frac{1}{1 - \exp\left[-\dfrac{C\,F(S^*_0)\,Pec_i\,S^*_0}{k}\right]}\right\} \tag{12}$$

and finally the results of (9) and (12) are in the sheet results S0=0.125.wms