0.1 The Eigenvalue Problem

$$\left.\begin{aligned}&\frac{d^2\Psi(x^*)}{dx^{*\,2}} - l^2\Psi(x^*) = 0, \quad \Psi(0) = 0\ , \Sigma(1) = 0,\\ &\frac{d^2\Sigma(x^*)}{dx^{*\,2}} - l^2\Sigma(x^*) = 0,\ \Psi(S_0^*) = \Sigma(S_0^*) + \Phi\, N_0, ,\end{aligned}\right\} \tag{1}$$

where

$$N_0 \quad = \quad F_1(S_0^*)\,\frac{C}{k}\,\frac{Pec_i}{R}\left\{(R-1) + \frac{k\,R\ R_3\,(1-R_1)}{C\,Pec_i}\right\}.$$

$$\begin{aligned}&F(S_0^*) = \frac{\dot{m}_{liq,vap}}{\dot{m}_i} = F_1(S_0^*)\,[1 - R\,G_1\,S_0^*]\,, \quad F_1(S_0^*) = \frac{1}{R\,S_0^* + 1 - S_0^*},\\ &G_1 = \frac{R_3\,k(1-R_1)}{C\,Pec_i}.\end{aligned} \tag{2}$$

The eigenvalue problem for the temperature distributions in the both phases,

$$\left.\begin{aligned}&\left(\frac{d^2}{dx^{*\,2}} - \omega_0\,\frac{d}{dx^*} - E_1\,\sigma - l^2\right)\phi_{liq} + \ \frac{d\Theta^0_{liq}}{dx^*}\,\frac{d\Psi}{dx^*} = 0,\\ &\phi_{liq}(0) = 0, \quad \phi_{liq}(S_0^*) = -\Phi\ \left.\frac{d\Theta^0_{liq}}{dx^*}\right|_{x^*=S_0^*},\\ &\left(\frac{d^2}{dx^{*\,2}} - \Omega_0\,\frac{d}{dx^*} - \frac{E_2\,k\,R_1}{C}\,\sigma - l^2\right)\phi_{vap} + \frac{R\,k}{C}\,\frac{d\Theta^0_{vap}}{dx^*}\,\frac{d\Sigma}{dx^*} = 0,\\ &\phi_{vap}(1) = 0, \quad \phi_{vap}(S_0^*) = -\Phi\ \left.\frac{d\Theta^0_{vap}}{dx^*}\right|_{x^*=S_0^*},\end{aligned}\right\} \tag{3}$$

where

$$\left.\begin{aligned}&\frac{d\Theta^0_{liq}}{dx^*} = N_1\ \exp\left(\omega_0\,x^*\right),\\ &\frac{d\Theta^0_{vap}}{dx^*} = N_2\ \exp\left(\Omega_0\,(x^*-1)\right),\end{aligned}\right\} \tag{4}$$

with leads to

$$\left.\frac{d\Theta^0_{liq}}{dx^*}\right|_{x^*=S_0^*} = N_3, \quad \text{and} \quad \left.\frac{d\Theta^0_{vap}}{dx^*}\right|_{x^*=S_0^*} = N_4, \tag{5}$$

where

$$\left.\begin{aligned} N_1 &= \frac{\omega_0}{\exp(\omega_0\, S_0^*) - 1}, \;\; N_2 = \frac{\Omega_0}{\exp(\Omega_0\,(S_0^* - 1)) - 1}, \\ N_3 &= N_1\, \exp(\omega_0\, S_0^*), \quad N_4 = N_2\, \exp\left(\Omega_0\,(S_0^* - 1)\right). \end{aligned}\right\} \tag{6}$$

and

$$\left.\begin{aligned} \omega_0 &= \frac{1}{k}\, Pec_i\, F(S_0^*)\, C, \\ \Omega_0 &= Pec_i\, F(S_0^*), \end{aligned}\right\} \tag{7}$$

0.2 Dispersion Analysis

An equation which represent the relationship between the decay rate σ and the wave number l is known as dispersion equation and can be obtained by using the energy jump across the liquid-vapour interface.

$$\varphi\,\sigma\, H_{liq}\,\Phi = \left\{\Phi\,\frac{d^2\Theta^0_{liq}}{dx^{*\,2}} + \frac{d\phi_{liq}}{dx^*}\right\} + \frac{\Theta_0}{k}\left\{\Phi\,\frac{d^2\Theta^0_{vap}}{dx^{*\,2}} + \frac{d\phi_{vap}}{dx^*}\right\} - H_{liq}\,\frac{d\Psi_1}{dx^*}. \tag{8}$$

From the basic state we know that

$$\left.\begin{aligned} \left.\frac{d^2\Theta^0_{liq}}{dx^{*\,2}}\right|_{x^*=S_0^*} &= N_3\,\omega_0, \\ \left.\frac{d^2\Theta^0_{vap}}{dx^{*\,2}}\right|_{x^*=S_0^*} &= N_4\,\Omega_0. \end{aligned}\right\} \tag{9}$$

Substituting (9) into (8) yields

$$\varphi\,\sigma\, H_{liq}\,\Phi = \left\{\frac{d\phi_{liq}}{dx^*} + \Phi\, N_3\,\omega_0\right\} + \frac{\Theta_0}{k}\left\{\frac{d\phi_{vap}}{dx^*} + \Phi\, N_4\,\Omega_0\right\} - H_{liq}\,\frac{d\Psi_1}{dx^*}. \tag{10}$$

The complete solution for $\sigma(l)$ can be obtained by substituting ϕ_{liq}, ϕ_{vap} and Ψ_1 into (10) which is solved numerically using Maple.