Products

```
> with(PauliAlgebra);
```

$$[Paravector, e0, e1, e2, e3] \quad (1.1)$$

```
> u := e0.<a,b,c>; #Convert Vector to Paravector during multiplication...
```

$$u := \left[0, \begin{bmatrix} a \\ b \\ c \end{bmatrix}\right] \quad (1.2)$$

```
> v := d*e1 + e*e2 + <0,0,f>; #...and addition
```

$$v := \left[0, \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right] \quad (1.3)$$

```
> u.v; #Geometric Product of Real Vectors = Dot Product + Wedge Product
```

$$\left[a\,d + b\,e + c\,f, \begin{bmatrix} \mathrm{I} f\,b - \mathrm{I}\,c\,e \\ -\mathrm{I} f\,a + \mathrm{I}\,c\,d \\ \mathrm{I}\,e\,a - \mathrm{I}\,b\,d \end{bmatrix}\right] \quad (1.4)$$

```
> (u.v)[S]; #Dot product
```

$$\left[a\,d + b\,e + c\,f, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right] \quad (1.5)$$

```
> (u.v)[V]/I; #Cross product
```

$$\left[0, \begin{bmatrix} b\,f - c\,e \\ -a\,f + c\,d \\ a\,e - b\,d \end{bmatrix}\right] \quad (1.6)$$

Rotations

```
> R := exp(Pi/4*e1.e2[B]); #Define a Pi/2 Rotor in the e1.e2 plane
```

$$R := \left[\frac{\sqrt{2}}{2}, \begin{bmatrix} 0 \\ 0 \\ -\frac{\mathrm{I}}{2}\sqrt{2} \end{bmatrix}\right] \quad (2.1)$$

```
> e1; #Unit vector in the x-axis
```

$$\left[0, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right] \quad (2.2)$$

```
> R.e1.R[D]; #Rotate e1 by Pi/2 in e1.e2 plane
```

$$\left[0, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right] \tag{2.3}$$

```
> R.R.e1.R[D].R[D]; #Perform the rotation twice on e1
```

$$\left[0, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}\right] \tag{2.4}$$

```
> R.e3.R[D]; #Unit vector in z direction is invariant
```

$$\left[0, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right] \tag{2.5}$$

Boosts

```
> B := exp(Pi/6*e1.e0[B]); #Define a Pi/3 boost (spacetime
  rotation) in e1.e0 plane
```

$$B := \left[\frac{e^{\frac{\pi}{6}}}{2} + \frac{e^{-\frac{\pi}{6}}}{2}, \begin{bmatrix} \frac{e^{\frac{\pi}{6}}}{2} - \frac{e^{-\frac{\pi}{6}}}{2} \\ 0 \\ 0 \end{bmatrix}\right] \tag{3.1}$$

```
> B := convert(B,trig);
```

$$B := \left[\cosh\left(\frac{\pi}{6}\right), \begin{bmatrix} \sinh\left(\frac{\pi}{6}\right) \\ 0 \\ 0 \end{bmatrix}\right] \tag{3.2}$$

```
> p := 5*e0 + 2*e1 + 3*e2+sqrt(2)*e3;; #Define a four momentum
```

$$p := \left[5, \begin{bmatrix} 2 \\ 3 \\ \sqrt{2} \end{bmatrix}\right] \tag{3.3}$$

```
> p.p[B]; #Square length is 5^2-2^2-3^2-2(mass^2)
```

$$\left[10, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right] \tag{3.4}$$

```
> B.p.B[D]; #Perform Lorentz Tranformation on four momentum p
```

$$\left[5\cosh\left(\frac{\pi}{3}\right) + 2\sinh\left(\frac{\pi}{3}\right), \begin{bmatrix} 2\cosh\left(\frac{\pi}{3}\right) + 5\sinh\left(\frac{\pi}{3}\right) \\ 3 \\ \sqrt{2} \end{bmatrix}\right] \tag{3.5}$$

Lorentz Transformations

```
> L := B.R; #Define a Lorentz rotor as a boost and rotation
```

$$L := \left[\frac{\cosh\left(\frac{\pi}{6}\right)\sqrt{2}}{2}, \left[\begin{matrix}\frac{\sqrt{2}\,\sinh\left(\frac{\pi}{6}\right)}{2}\\ -\frac{\sqrt{2}\,\sinh\left(\frac{\pi}{6}\right)}{2}\\ -\frac{I}{2}\cosh\left(\frac{\pi}{6}\right)\sqrt{2}\end{matrix}\right]\right] \tag{4.1}$$

```
> pnew := L.p.L[D]; #Perform a rotation and boost on four
  momentum
```

$$pnew := \left[5\cosh\left(\frac{\pi}{3}\right) - 3\sinh\left(\frac{\pi}{3}\right), \left[\begin{matrix}-3\cosh\left(\frac{\pi}{3}\right) + 5\sinh\left(\frac{\pi}{3}\right)\\ 2\\ \sqrt{2}\end{matrix}\right]\right] \tag{4.2}$$

```
> evalf[4](pnew);
```

$$\left[4.253, \left[\begin{matrix}1.445\\ 2.\\ 1.414\end{matrix}\right]\right] \tag{4.3}$$

```
> p.p[B]; pnew.pnew[B]; #Square length is invariant
```

$$\left[10, \left[\begin{matrix}0\\ 0\\ 0\end{matrix}\right]\right]$$

$$\left[10, \left[\begin{matrix}0\\ 0\\ 0\end{matrix}\right]\right] \tag{4.4}$$