```
> restart;
> interface(rtablesize = infinity):
> with(LinearAlgebra) : with(RootFinding) : with(ArrayTools) :
> Phi := C1·cos(beta·x) + C2·sin(beta·x) + C3·cosh(beta·x) + C4·sinh(beta·x);
```

$$\Phi := C1\cos(\beta x) + C2\sin(\beta x) + C3\cosh(\beta x) + C4\sinh(\beta x) \tag{1}$$

```
> bc1 := EI·diff(Phi, x$2) − Kr·diff(Phi, x) : x := 0; BC1 := bc1; x := 'x':
```

$$x := 0$$

$$BC1 := EI\left(-C1\,\beta^2 + C3\,\beta^2\right) - Kr\left(C2\,\beta + C4\,\beta\right) \tag{2}$$

```
> bc2 := EI·diff(Phi, x$3) + Kt·Phi : x := 0; BC2 := bc2; x := 'x':
```

$$x := 0$$

$$BC2 := EI\left(-C2\,\beta^3 + C4\,\beta^3\right) + Kt\left(C1 + C3\right) \tag{3}$$

```
> bc3 := diff(Phi, x$2) : x := L; BC3 := bc3/β^2 ; x := 'x':
```

$$x := L$$

$$BC3 := \frac{-C1\,\beta^2\cos(\beta L) - C2\,\beta^2\sin(\beta L) + C3\,\beta^2\cosh(\beta L) + C4\,\beta^2\sinh(\beta L)}{\beta^2} \tag{4}$$

```
> bc4 := diff(Phi, x$3) : x := L; BC4 := bc4/β^3 ; x := 'x':
```

$$x := L$$

$$BC4 := \frac{C1\,\beta^3\sin(\beta L) - C2\,\beta^3\cos(\beta L) + C3\,\beta^3\sinh(\beta L) + C4\,\beta^3\cosh(\beta L)}{\beta^3} \tag{5}$$

```
> a11 := collect(BC1, C1): a12 := collect(BC1, C2): a13 := collect(BC1, C3): a14 :=
    collect(BC1, C4):
> a21 := collect(BC2, C1): a22 := collect(BC2, C2): a23 := collect(BC2, C3): a24 :=
    collect(BC2, C4):
> a31 := collect(BC3, C1): a32 := collect(BC3, C2): a33 := collect(BC3, C3): a34 :=
    collect(BC3, C4):
> a41 := collect(BC4, C1): a42 := collect(BC4, C2): a43 := collect(BC4, C3): a44 :=
    collect(BC4, C4):
```

$$
> A11 := \frac{select(has, a11, C1)}{C1} : A12 := \frac{select(has, a12, C2)}{C2} : A13 := \frac{select(has, a13, C3)}{C3} :
$$

$$
A14 := \frac{select(has, a14, C4)}{C4} :
$$

$$
> A21 := \frac{select(has, a21, C1)}{C1} : A22 := \frac{select(has, a22, C2)}{C2} : A23 := \frac{select(has, a23, C3)}{C3} :
$$

$$
A24 := \frac{select(has, a24, C4)}{C4} :
$$

$$
> A31 := \frac{select(has, a31, C1)}{C1} : A32 := \frac{select(has, a32, C2)}{C2} : A33 := \frac{select(has, a33, C3)}{C3} :
$$

$$
A34 := \frac{select(has, a34, C4)}{C4} :
$$

```maple
> A41 := select(has, a41, C1)/C1 : A42 := select(has, a42, C2)/C2 : A43 := select(has, a43, C3) :
    A44 := select(has, a44, C4)/C4 :
```

```maple
> M := Matrix([[A11, A12, A13, A14], [A21, A22, A23, A24], [A31, A32, A33, A34], [A41, A42,
    A43, A44]]);
```

$$M := \begin{bmatrix} -EI\,\beta^2 & -Kr\,\beta & EI\,\beta^2 & -Kr\,\beta \\ Kt & -EI\,\beta^3 & Kt & EI\,\beta^3 \\ -\cos(\beta L) & -\sin(\beta L) & \cosh(\beta L) & \sinh(\beta L) \\ \sin(\beta L) & -\cos(\beta L) & \sinh(\beta L) & \cosh(\beta L) \end{bmatrix}$$ (6)

```maple
> detM := simplify(Determinant(M));
```

$$detM := -2\,\Big(\big(\big(EI^2\,\beta^4 - Kr\,Kt\big)\cos(\beta L) + EI\,\beta\,\sin(\beta L)\,\big(Kr\,\beta^2 + Kt\big)\big)\cosh(\beta L)$$ (7)
$$+\,EI\,\beta\,\sinh(\beta L)\,\big(Kr\,\beta^2 - Kt\big)\cos(\beta L) - EI^2\,\beta^4 - Kr\,Kt\Big)\,\beta$$

```maple
> 
```

```maple
> EI := 20·10^9·7243.32; L := 121; Kt := 4.5·10^9; Kr := 5.7·10^9; rho := 400; A := 30;
```

$$EI := 1.448664000\,10^{14}$$
$$L := 121$$
$$Kt := 4.500000000\,10^9$$
$$Kr := 5.700000000\,10^9$$
$$\rho := 400$$
$$A := 30$$ (8)

```maple
> 
```

```maple
> B := Analytic(detM = 0, beta, re = 0..0.1, im = 0..0.0001);
```

$$B := 0.0435503148890837,\ 0.\,I,\ 0.0279531203897676,\ 0.00285611175750400,$$ (9)
$$0.0659083580987740,\ 0.0912286729053350$$

```maple
> β_f := remove(has, [B], I);
```

$$\beta_f := [0.0435503148890837,\ 0.0279531203897676,\ 0.00285611175750400,$$ (10)
$$0.0659083580987740,\ 0.0912286729053350]$$

```maple
> β_omega := sort(β_f);
```

$$\beta_\omega := [0.00285611175750400,\ 0.0279531203897676,\ 0.0435503148890837,$$ (11)
$$0.0659083580987740,\ 0.0912286729053350]$$

```maple
> for i from 1 by 1 to Size(β_omega, 2) do beta := β_omega[i] : evalf(detM); end do;
```

$$\beta := 0.00285611175750400$$
$$-1.085322468\,10^9$$
$$\beta := 0.0279531203897676$$
$$8.385936116\,10^{12}$$

$$\beta := 0.0435503148890837$$

$$-1.829113225 \; 10^{14}$$

$$\beta := 0.0659083580987740$$

$$4.534495038 \; 10^{15}$$

$$\beta := 0.0912286729053350$$

$$1.219052265 \; 10^{19} \tag{12}$$

> omega $:= RandomVector\left(Size\left(\beta_{\mathrm{omega}}, 2\right)\right):$

> **for** i **from** 1 **by** 1 **to** $Size\left(\beta_{\mathrm{omega}}, 2\right)$ **do** omega$[i] := \mathrm{sqrt}\left(\dfrac{\left(\beta_{\mathrm{omega}}[i]\right)^4 \cdot EI}{\mathrm{rho} \cdot A} \right)$ **end do**;

$$\omega_1 := 0.8962797930$$

$$\omega_2 := 85.85266893$$

$$\omega_3 := 208.3894891$$

$$\omega_4 := 477.2810552$$

$$\omega_5 := 914.4414958 \tag{13}$$

> $f := \left(\dfrac{\mathrm{omega}}{2 \cdot \mathrm{Pi}} \right);$

$$f := \begin{bmatrix} 0.142647359400000 \\ 13.6638766350000 \\ 33.1662172700000 \\ 75.9616391500000 \\ 145.537884200000 \end{bmatrix} \tag{14}$$

>

>