Solve for $U_0(y_1,y_2)$, $U_1(y_1,y_2)$, $U_2(y_1,y_2)$, **(all** $[0,\infty)\times[0,\infty)\mapsto[0,\infty)$**)**
(and afterwards, I need a nice plot of level curves, $-\vec{q}(y_1,y_2)$, $-\vec{Q}(y_1,y_2)$**):**
Constants, definitions, PDEs, algebraic equations (AEsomething), BCs. "Everything" are functions to be evaluated at $\vec{y}=\binom{y_1}{y_2}\in[0,\infty)^2$.

$$\text{assume}\quad \vec{\mu}=\binom{\mu_1}{\mu_2}\in(0,2)\times(0,2),\quad \mathbf{M}=\begin{bmatrix}2&-1\\-1&2\end{bmatrix} \tag{cnst}$$

$$\vec{F}(\vec{y}):\quad [0,\infty)\times[0,\infty)\ni\vec{y}\longrightarrow\frac{1}{2\pi}\begin{pmatrix}\arctan y_1\\ \arctan y_2\end{pmatrix} \tag{df$F$}$$

`%` F_i `given permutation-invariant function, threw in one`

$$U_0=\vec{F}^\top\big(\mathbf{M}^{-1}\vec{q}-\vec{r}\big)-\frac{1}{4}\Big\|\mathbf{M}^{-1}\vec{q}-\vec{r}\Big\|^2-\frac{1}{2}\big(2\vec{F}+\vec{r}-\vec{s}\big)^\top\mathbf{M}^{-1}\vec{q} \tag{PDE0}$$

$$U_1=\big(r_1+q_2\big)F_1-\frac{1}{4}\big(r_1+q_2\big)^2-\frac{\partial U_1}{\partial y_2} \tag{PDE1}$$

$$U_2=\big(r_2+q_1\big)F_2-\frac{1}{4}\big(r_2+q_1\big)^2-\frac{\partial U_2}{\partial y_1} \tag{PDE2}$$

$$\vec{r}(\vec{y})=\begin{pmatrix}\partial U_1/\partial y_1\\ \partial U_2/\partial y_2\end{pmatrix}\qquad \texttt{\% could insert, only for readability} \tag{AEr}$$

$$\vec{s}(\vec{y})=2\vec{F}+\vec{r}-2\mathbf{M}\big(\vec{\mu}-\vec{1}+2\vec{F}+\vec{\nabla}U_0\big)\qquad \texttt{\% ditto} \tag{AEs}$$

$$\vec{q}(\vec{y})=\text{coordwiseMax}\Big\{\vec{0},\ \mathbf{M}\vec{s}+\tfrac{9}{5}\mathbf{M}^2\text{coordwiseMax}\big\{\vec{0},-\mathbf{M}\vec{s}\big\}\Big\} \tag{AEq}$$

$$0=U_0(y_1,0),\qquad 0=U_0(0,y_2) \tag{BCs0}$$

$$0=U_1(y_1,0),\qquad 0=U_1(0,y_2) \tag{BCs1}$$

$$0=U_2(y_1,0),\qquad 0=U_2(0,y_2) \tag{BCs2}$$

where $\vec{0}=\binom{0}{0}$, $\vec{1}=\binom{1}{1}$ and where "coordwiseMax" should be self-explanatory. Will also need in the plot:

$$\vec{Q}=\vec{F}-\frac{1}{2}\Big(\vec{r}+\begin{bmatrix}0&1\\1&0\end{bmatrix}\vec{q}\Big)\qquad \texttt{\% want } -\vec{q},\ -\vec{Q}\texttt{ as arrows in plot} \tag{Q}$$