```
> # http://www.mapleprimes.com/questions/208551-Eigenvalues-And-Characteristic--
  Polynomial
> restart; interface(version); Digits := 10;
  with(LinearAlgebra): #with(Student[NumericalAnalysis]);
```

Standard Worksheet Interface, Maple 18.02, Windows 7, October 8 2014 Build ID 987602

$$Digits := 10 \quad \textbf{(1)}$$

```
> M := Matrix(8, 8, {(1, 1) = 11.90787610, (1, 2) = 4, (1, 3) = -1, (1, 4) = 1,
  (1, 5) = 1, (1, 6) = 5, (1, 7) = -1, (1, 8) = 1, (2, 1) = 4, (2, 2) =
  19.70552151, (2, 3) = -1, (2, 4) = 2, (2, 5) = 1, (2, 6) = 4, (2, 7) = -1, (2,
  8) = 2, (3, 1) = -1, (3, 2) = -1, (3, 3) = 30.54549819, (3, 4) = 3, (3, 5) = 1,
  (3, 6) = 3, (3, 7) = -1, (3, 8) = 3, (4, 1) = 1, (4, 2) = 2, (4, 3) = 3, (4, 4)
  = 40.06265749, (4, 5) = 1, (4, 6) = 2, (4, 7) = -1, (4, 8) = 4, (5, 1) = 1, (5,
  2) = 1, (5, 3) = 1, (5, 4) = 1, (5, 5) = 51.58714029, (5, 6) = 1, (5, 7) = -1,
  (5, 8) = 5, (6, 1) = 5, (6, 2) = 4, (6, 3) = 3, (6, 4) = 2, (6, 5) = 1, (6, 6)
  = 64.70213143, (6, 7) = -1, (6, 8) = 6, (7, 1) = -1, (7, 2) = -1, (7, 3) = -1,
  (7, 4) = -1, (7, 5) = -1, (7, 6) = -1, (7, 7) = 70.17067582, (7, 8) = 7, (8, 1)
  = 1, (8, 2) = 2, (8, 3) = 3, (8, 4) = 4, (8, 5) = 5, (8, 6) = 6, (8, 7) = 7,
  (8, 8) = 71.31849917});
```

$$M := \begin{bmatrix} [11.90787610, 4, -1, 1, 1, 5, -1, 1], \\ [4, 19.70552151, -1, 2, 1, 4, -1, 2], \\ [-1, -1, 30.54549819, 3, 1, 3, -1, 3], \\ [1, 2, 3, 40.06265749, 1, 2, -1, 4], \\ [1, 1, 1, 1, 51.58714029, 1, -1, 5], \\ [5, 4, 3, 2, 1, 64.70213143, -1, 6], \\ [-1, -1, -1, -1, -1, -1, 70.17067582, 7], \\ [1, 2, 3, 4, 5, 6, 7, 71.31849917] \end{bmatrix} \quad \textbf{(2)}$$

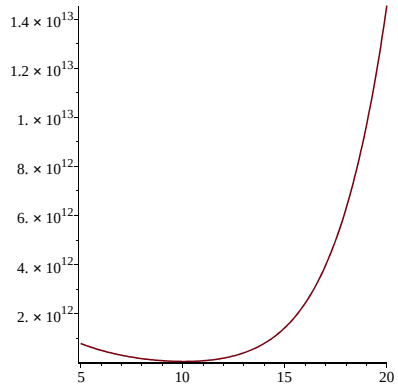
```
> E := 'Eigenvalues(M)';
```

$$E := LinearAlgebra:-Eigenvalues(M) \quad \textbf{(3)}$$

```
> p := unapply(CharacteristicPolynomial(M, x), x);
```

$$p := x \to 4.0319999995768881\ 10^{12} - 1.0958399992996265\ 10^{12} x + 235.29233460338537\ x^7 + 538.5674907870567\ x^8 - 4.537012841811221\ 10^6 x^5 + 54968.919372139746\ x^6 - 6.728400085950639\ 10^9 x^3 + 2.2448876139292035\ 10^8 x^4 + 1.1812400036410367\ 10^{11} x^2 \quad \textbf{(4)}$$

```
> plot(p, Re(E[1]/2) .. Re(E[1]*2));
  minimize(p(x), x=  Re(E[1]/2) .. Re(E[1]*2) ): minimum = %;
  fsolve(D(p)(x), x=  Re(E[1]/2) .. Re(E[1]*2) );
```



$$minimum = 5.988547358\ 10^{10}$$

$$10.05178029 \quad \textbf{(5)}$$

```
> n:= RowDimension(M);
  det:= Determinant(M); # det:= fnormal(det);
  s:= 1/abs(det)^(1/n); # scaling
```

$$n := 8$$

$$det := 4.03199999980081\ 10^{12}$$

$$s := 0.0265650069931895 \quad \textbf{(6)}$$

```
> MM:='s*M'; ``=%;
  'Determinant(MM)': '%'=%;
```

$$MM := s\,M$$

$$= \begin{bmatrix} [0.316332811870534, 0.106260027972758, -0.0265650069931895, 0.0265650069931895, \\ 0.0265650069931895, 0.132825034965947, -0.0265650069931895, 0.0265650069931895 \\ ], \\ [0.106260027972758, 0.523477316717595, -0.0265650069931895, \end{bmatrix}$$

0.0531300139863789, 0.0265650069931895, 0.106260027972758,
−0.0265650069931895, 0.0531300139863789],
[−0.0265650069931895, −0.0265650069931895, 0.811441373027806,
0.0796950209795684, 0.0265650069931895, 0.0796950209795684,
−0.0265650069931895, 0.0796950209795684],
[0.0265650069931895, 0.0531300139863789, 0.0796950209795684, 1.06426477638760,
0.0265650069931895, 0.0531300139863789, −0.0265650069931895, 0.106260027972758
],
[0.0265650069931895, 0.0265650069931895, 0.0265650069931895,
0.0265650069931895, 1.37041274256250, 0.0265650069931895, −0.0265650069931895,
0.132825034965947],
[0.132825034965947, 0.106260027972758, 0.0796950209795684, 0.0531300139863789,
0.0265650069931895, 1.71881257391221, −0.0265650069931895, 0.159390041959137],
[−0.0265650069931895, −0.0265650069931895, −0.0265650069931895,
−0.0265650069931895, −0.0265650069931895, −0.0265650069931895,
1.86408449387513, 0.185955048952326],
[0.0265650069931895, 0.0531300139863789, 0.0796950209795684, 0.106260027972758,
0.132825034965947, 0.159390041959137, 0.185955048952326, 1.89457642919483]]

$$LinearAlgebra{:}\text{-}Determinant(MM) = 1.00000000000000 \quad \textbf{(7)}$$

```
> EE := 'Eigenvalues(MM)';
```

$$EE := LinearAlgebra{:}\text{-}Eigenvalues(MM) \quad \textbf{(8)}$$

```
> #(sigma*B).v=(sigma*lambda)*v;
  #eval(%, sigma=1); # the same as dividing by sigma, if sigma <> 0
> #'1/s*EE = E + smallError'; %;
  'E[1] = EE[1]/s + smallError'; %;
```

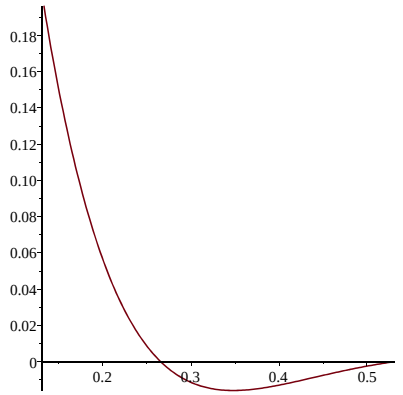
$$E_1 = \frac{EE_1}{s} + smallError$$

$$9.99999999808026 + 0.\,\mathrm{I} = 9.99999999808027 + 0.\,\mathrm{I} + smallError \quad \textbf{(9)}$$

```
> CharacteristicPolynomial(MM, x): expand(%): sort(%):
  pp:= unapply(%, x);
```

$$pp := x \to x^8 - 9.563402517548205\,x^7 + 38.5311979715982\,x^6 - 85.03600515852573\,x^5 + 111.79869604940745\,x^4 - 89.01464722379629\,x^3 + 41.514303152788074\,x^2 - 10.230967165486561\,x + 1.0000000000000018 + 0.0\,\mathrm{I} \quad \textbf{(10)}$$

```
> plot(pp, Re(EE[1]/2) .. Re(EE[1]*2));
```




```
> for k to 8 do
    pp(EE[k])
  end do;
```

$$-2.22044604925031\ 10^{-16} + 0.\,\mathrm{I}$$
$$-8.88178419700125\ 10^{-16} + 0.\,\mathrm{I}$$
$$-8.70414851306123\ 10^{-14} + 0.\,\mathrm{I}$$
$$-2.29150032282632\ 10^{-13} + 0.\,\mathrm{I}$$
$$-5.32907051820075\ 10^{-15} + 0.\,\mathrm{I}$$
$$6.57252030578093\ 10^{-14} + 0.\,\mathrm{I}$$
$$-7.46069872548105\ 10^{-14} + 0.\,\mathrm{I}$$
$$-1.06581410364015\ 10^{-14} + 0.\,\mathrm{I} \quad \textbf{(11)}$$