Particular Case

$E =$ const
$\rho =$ const] homogeneous

$A =$ const
$I =$ const] uniform beam

differential eq: $\frac{\partial^2}{\partial x^2}\left[EI \frac{\partial^2 w}{\partial x^2}\right] + \rho A \frac{\partial^2 w}{\partial t^2} = 0$

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0$$

$$w(x,t) = \underbrace{W(x)}_{} \; \underbrace{\cos(\omega_n t + \phi)}_{}$$

function in space — sought — $\boxed{\text{mode shape}}$

harmonic function

$\omega =$ sought natural freq.

$$EI \frac{\partial^4}{\partial x^4}\left[W(x)\cos \omega t\right] + \rho A \frac{\partial^2}{\partial t^2}\left[W(x)\cos \omega t\right]$$

$$\cos(\omega t)\, EI \frac{d^4 W}{dx^4} + \underbrace{\rho A\, W(x) \frac{d^2 \cos(\omega t)}{dt^2}}_{-\rho A W(x)\omega^2 \cos(\omega t)}$$

derivative

$$\boxed{EI \frac{d^4 W}{dx^4} - \rho A \omega^2 W = 0}$$

ODE with const co
divide by EI

$$\frac{d^4 W}{dx^4} - \underbrace{\frac{\rho A \omega^2}{EI}}_{\alpha^4} W = 0$$

Q What do we want

A Find ω such that $W(x) \neq 0$

shape is not zero

mode shape is automatically not zero

$W = Ce^{rx}$

$Cr^4 e^{rx} - \alpha^4 Ce^{rx} = 0$

$Ce^{rx}(r^4 - \alpha^4) = 0$

$C \neq 0$

$r^4 - \alpha^4 = 0$

$(r^2)^2 - (\alpha^2)^2$

$(r^2 + \alpha^2)(r^2 - \alpha^2) = 0$

$r_{1,2} = \pm i\alpha$

$r_{3,4} = \pm \alpha$

~~$W(x) = C_1(\cos\alpha x + i\sin\alpha x)$~~

$W(x) = C_1 e^{i\alpha x} + C_2 e^{-i\alpha x}$

$C_3 e^{\alpha x} + C_4 e^{-\alpha x}$

$W(x) = C_1(\cos\alpha x + i\sin\alpha x)$

$C_2(\cos(\alpha x) - i\sin(\alpha x))$

$+ C_3 e^{\alpha x} + C_4 e^{-\alpha x}$

$$W(x) = \sin\alpha x\,(iC_1 - iC_2) + \cos\alpha x\,(C_1 + C_2) + C_3 e^{\alpha x} + C_4 e^{-\alpha x}$$

$$W(x) = D_1 \sin(\alpha x) + D_2 \cos(\alpha x) + C_3 e^{\alpha x} + C_4 e^{-\alpha x}$$

$$W(x) = D_1 \sin(\alpha x) + D_2 \cos(\alpha x) + C_3 e^{\alpha x} + C_4 e^{-\alpha x}$$

$$\cosh(\alpha x) = \frac{e^{\alpha x} + e^{-\alpha x}}{2}$$

$$\sinh(\alpha x) = \frac{e^{\alpha x} - e^{-\alpha x}}{2}$$

ADD ;

$$e^{\alpha x} = \sinh(\alpha x) + \cosh(\alpha x)$$

subtract ;

$$e^{-\alpha x} = \cosh(\alpha x) - \sinh(\alpha x)$$

$$W(x) = D_1 \sin(\alpha x) + D_2 \cos(\alpha x)$$
$$+ C_3 \left[\sinh(\alpha x) + \cosh(\alpha x) \right]$$
$$+ C_4 \left[\cosh(\alpha x) - \sinh(\alpha x) \right]$$

$$W(x) = D_1 \sin(\alpha x) + D_2 \cos(\alpha x)$$
$$+ \sinh(\alpha x) \left[C_3 - C_4 \right] \rightarrow D_3$$
$$+ \cosh(\alpha x) \left[C_3 + C_4 \right] \rightarrow D_4$$

$$\boxed{W(x) = D_1 \sin(\alpha x) + D_2 \cos(\alpha x) + D_3 \sinh(\alpha x) + D_4 \cosh(\alpha x)}$$

General form for mode shape.

Satisfy B.C so that $W(x) \neq 0$

or

$$D_1^2 + D_2^2 + D_3^2 + D_4^2 \neq 0$$

and find α

If we know α then $\frac{\rho A \omega^2}{EI} = \alpha^4 \times L^4$

$$\frac{\rho A \omega^2}{EI} L^4 = (\alpha L)^4$$

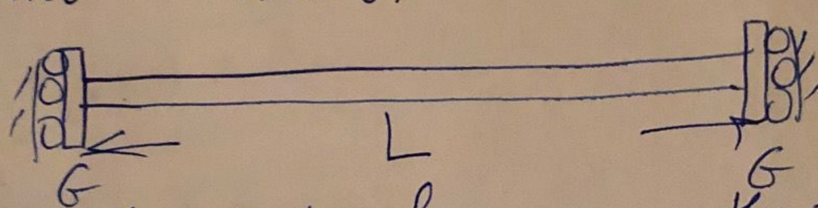
once αL is known we can find $\omega^2 = \frac{EI}{\rho A L^4} (\alpha L)^4$

PROJECT FALL 2018

Vibrations

Student Name: MR. KHAN
Lecturer: Professor I. Elishakoff

Topic: FREE VIBRATION OF A BEAM CARRYING MASS WITH GUIDED ENDS



1) please check if this beam possess the zero natural frequency.
2) For nonzero frequencies, please derive transcendental equation, solve it on Maple, and find first to nonzero natural frequencies.

References:

Lecture notes.

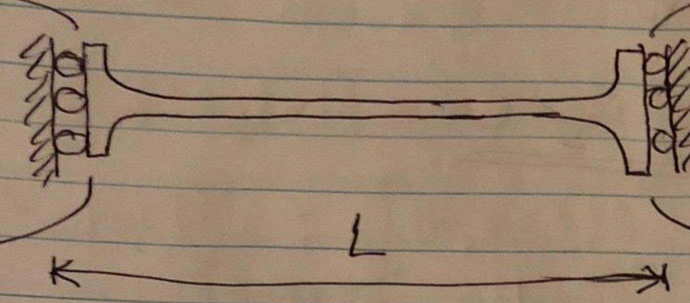
Comments:
1.Please consult the document "Format" for the way of submission of the project. 2. See back side for possible details.3. This page or its scan should be included in the project submission.

Date of Assignment:
Date of Completion:

Free Vibrations of a Beam carrying mass with guided ends.

$\frac{dw}{dx} = 0$

$\frac{d^3 w}{dx^3} = 0$



$\frac{dW}{dx} = 0$

$\frac{d^3 W}{dx} = 0$

at $x = 0$

$W'(0) = 0$

$W'''(0) = 0$

at $x = L$

$w'(L) = 0$

$w'''(L) = 0$

$$W(x) = [A_1 K_1(\alpha x) + A_2 K_2(\alpha x) + A_3 K_3(\alpha x) + A_4 K_4(\alpha x)]$$

$$W'(x) = \alpha[A_1 K_4(\alpha x) + A_2 K_1(\alpha x) + A_3 K_2(\alpha x) + A_4 K_3(\alpha x)]$$

$$W''(x) = \alpha^2 [A_1 K_3(\alpha x) + A_2 K_2(\alpha x) + A_3 K_1(\alpha x) + A_4 K_4(\alpha x)]$$

$$W'''(x) = \alpha^3 [A_1 K_2(\alpha x) + A_2 K_3(\alpha x) + A_3 K_4(\alpha x) + A_4 K_1(\alpha x)]$$

$\Rightarrow$ 1st B.C

$$W'(0) = 0 = A_1 K_4(0) + A_2 K_1(0) + A_3 K_2(0) + A_4 K_3(0)$$

$K_1(0) = 1 \quad K_2(0) = 0 \quad K_3(0) = 0 \quad K_4(0) = 0$

therefore

$$0 = A_2$$

$\Rightarrow$ 2nd B.C;

$$W'''(0) = 0 = \alpha^3 [A_1 K(\alpha x) + A_2 K(\alpha x) + A_3 K(\alpha x) + A_4 K(\alpha x)]$$

$$W'''(0) = 0 = \alpha^3\left[A_1K_2(0) + A_2K_3(0) + A_3K_4(0) + A_4K_1(0)\right]$$

$$0 = A_4$$

Hence $A_4 = A_2 = 0$

$\Rightarrow$ 3rd B.C

$$W'(L) = 0 = \alpha\left[A_1K_4(\alpha L) + \cancelto{0}{A_2K_1(\alpha L)} + A_3K_2(\alpha L) + \cancelto{0}{A_4K_3(\alpha L)}\right]$$

$$0 = \alpha\left[A_1K_4(\alpha L) + A_3K_2(\alpha L)\right]$$

$\alpha \neq 0$

therefore

$$0 = A_1K_4(\alpha L) + A_3K_2(\alpha L)$$

$\Rightarrow$ 4th B.C

$$W'''(L) = 0 = \alpha^3\left[A_1K_2(\alpha L) + A_3K_4(\alpha L)\right]$$

$\alpha \neq 0$

therefore

$$0 = A_1K_2(\alpha L) + A_3K_4(\alpha L)$$

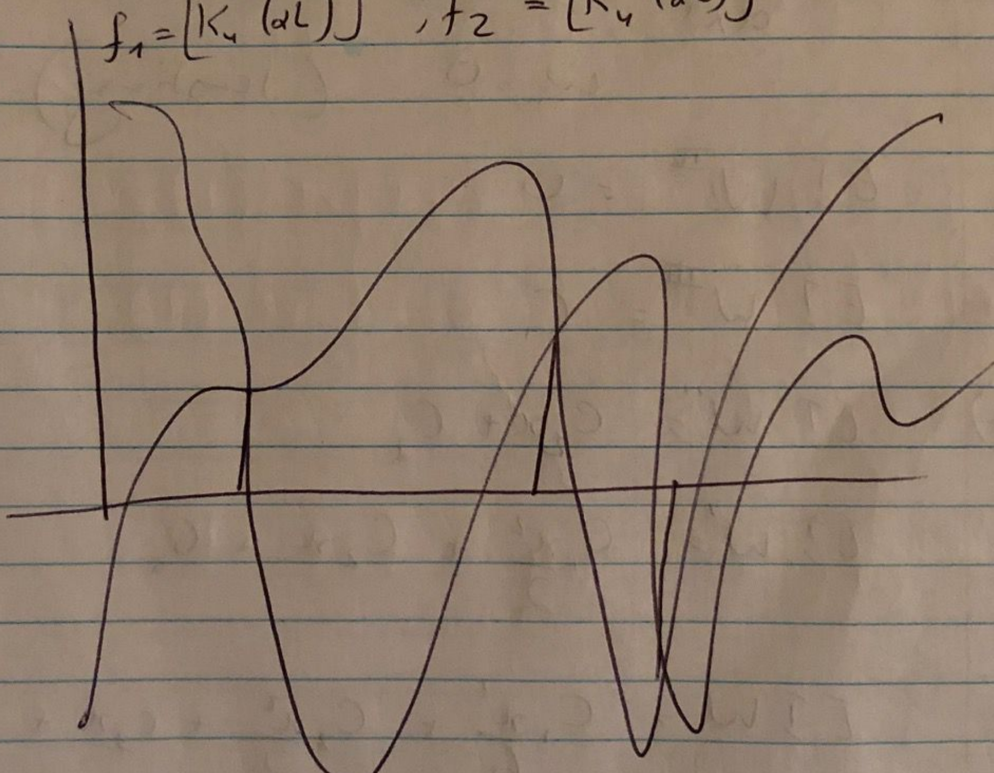
$$\begin{vmatrix} K_4(\alpha L) & + & K_2(\alpha L) \\ K_2(\alpha L) & + & K_4(\alpha L) \end{vmatrix} = 0$$

$$K_4(\alpha L) . K_4(\alpha L) - K_2(\alpha L) . K_2(\alpha L) = 0$$

$$\underbrace{\left[K_4(\alpha L)\right]^2 - \left[K_2(\alpha L)\right]^2}_{\text{simplify}} = 0$$ characteristic equation

$$K_4^2(\alpha L) = K_2^2(\alpha L)$$

$$f_1 = \left[K_4(\alpha L)\right]^2 , f_2 = \left[K_4(\alpha L)\right]^2$$



graphically