(b) The following alternating series converges to the value shown (the sum of the series).

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+2)n!} = 1 - \frac{2}{e}.$$

Show this by considering the integral

$$\int xe^{-x}\,dx = -xe^{-x} - x.$$

Also study this series numerically using the code from Example 3.3. Compute the partial sums s_k of this series until k is the first value for which $a_{k+1} \le \varepsilon$, with $\varepsilon = 10^{-9}$. What is the value of $k = k_0$ for which this occurs? Can you predict this value k_0 in advance? Print out a table of the values of the s_k's, four values to a line. Does this series converge very rapidly? Why?

4. **(Approximating Lengths of Curves)** Write a Maple procedure to compute the approximate length for a curve that is the graph of a function $f : I \to \mathbb{R}$ defined on a closed interval $I = [a, b]$. Recall that the graph of f is the set of points

$$C = \{\, (x, f(x)) \,|\, x \in R \},$$

in $\mathbb{R}^2$. Here are some instructions for designing your procedure: Approximate the curve C by a polygon for which you can calculate the lengths of its individual sides. Construct this polygon as follows. Partition I into subintervals by using the standard partitions of $[a, b]$ into N subintervals of equal lengths. Then the endpoints of the subintervals in the partition of I are $x_i = a+(b-a)i/N$, for $i = 0, \ldots, N$. The ith subinterval is $I_i = [x_{i-1}, x_i]$, for $i = 1, \ldots, N$. Corresponding to this subdivision is a polygonal curve P_N in $\mathbb{R}^2$ with vertices $Q_i = (x_i, y_i)$, $i = 1, \ldots, N$, where $y_i = f(x_i)$. Connecting these points with straight line segments produces a polygonal approximation to the curve C. See Figure 3.11. Your procedure should compute the approximating length to be the sum of the lengths of all the sides in this polygon, i.e.,

$$AL(f) = \sum_{i=1}^{N} d(Q_{i-1}, Q_i), \tag{3.2}$$

where $d(Q_{i-1}, Q_i)$ denotes the length of the ith side of the polygon. Here are some further directions and parts to the exercise.

(a) The distance formula gives

$$d(Q_{i-1}, Q_i) = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}, \tag{3.3}$$

where $\Delta y_i = y_i - y_{i-1}$. This is the length of the ith side of the approximating polygon. Use this in your procedure.