5. **(Infinite Series and Products):** This exercise deals with elementary programming constructs involving do loops, Maple's `seq` command, and some graphical display. The mathematical topics covered are infinite series and infinite products (the latter might be new to you).

Infinite Series: For a series $\sum_{n=1}^{\infty} a_n$, the convergence or divergence of the series, i.e., the question of whether we can sum the infinitely many terms in the series, is, by definition, dependent upon whether the sequence $\{s_n\}_{n=1}^{\infty}$ of partial sums converges or diverges (i.e., $\lim_{n\to\infty} s_n$ exists or not). This is a theoretical question and *cannot* be determined by a computer. However, if we can show, using one of the many "tests" for convergence, that the series converges, then the computer can be used to estimate the value for the sum of the series. To explore this do the following.

(a) The standard p-series is the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^p},$$

with $p > 0$. Do an experimental study of this type of series for $p = 1, 2, 3$. Specifically:

(i) Define the terms a_i of the series (which also depend on p) as a function of two variables `a:=(n,p)-> 1/n^p`. Use a two-dimensional array `s` and do loops to calculate the partial sums s_n for $n = 1, \ldots, 180$ and $p = 1, 2, 3$. *Note*: use an `evalf` to force floating-point evaluation (otherwise Maple will give you fractions).

(ii) For $p = 1, 2, 3$, display in your worksheet the values of `s[n,p]` for $n = 100, \ldots, 180$, with five values per line. You can do this with a do loop and the `seq` command.

(iii) For $p = 1, 2, 3$, build a list

```
L[p]:=[[1,s[1,p]], . . . , [180,s[180,p]]]
```

of points `[n,s[n,p]]` for plotting. Build a plot structure `ps[p]` using the plot command

```
ps[p]:=plot(L[p],style=point,color=c[p]):
```

Here `c` is an array of your favorite three colors. Display all three plots in the same picture. Annotate this picture with the values of p and your best guess of the values $\lim_{n\to\infty} s_n$. For this, study the values displayed in part (b). How many decimal places of accuracy are certain?

(b) Do a study, similar to that in part (a), for the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}.$$