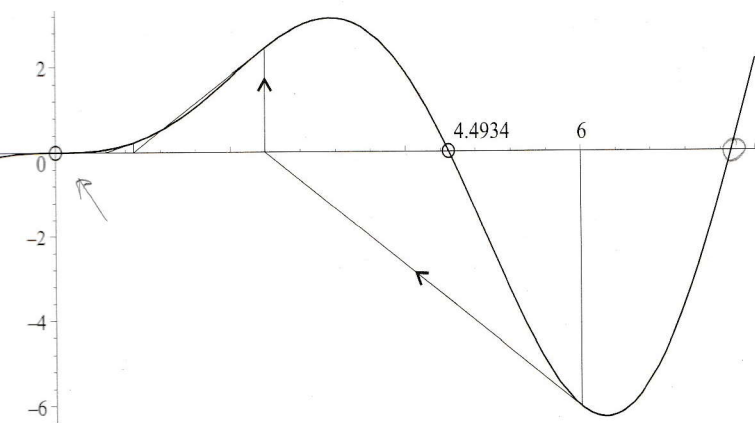




10: For initial approximation $x_0 = 6$, Newton's method converges to $r = 0$.

Determine how we got this. **Hint:** First show that

$$\int \ln(1+x) dx = (1+x) \ln(1+x) - x.$$

Then expand the integrand in a Maclaurin series. In addition, choose one (or more, if you wish) of the following alternating series to study as in Example 3.3. Determine the exact sum of the series by methods as suggested in the above hint.

a) The alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots,$$

is a famous alternating series. Use the code in Example 3.3 to compute the partial sums s_k of this series until k is the first value for which $a_{k+1} \leq \epsilon$, with $\epsilon = .001$. What is the value of $k = k_0$ for which this occurs? Can you predict this value k_0 in advance? Produce a table of the last 100 values of the partial sums s_k , $k = k_0 - 100, \dots, k_0$, with five values per line. Does this series converge very slowly? Why?

Show that the series converges to $\pi/4$,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}.$$

Hint: Use a method like that in the preceding exercise, but now with

$$\int \frac{1}{1+x^2} dx = \tan^{-1}(x).$$