

- (c) Do a study, similar to that in part (a), for the logarithmic series

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(\ln(n+1))^p}.$$

Here, to save space, omit the tables of values, and just use graphical displays as in Part (a)(iii). Click on the graphs to estimate the values of the sums for the series. Also use the integral test to determine, in general, for what values of  $p > 0$  the series converges (hand in this work too).

- (d) Do a study, similar to that in part (a), for the series

$$\sum_{n=1}^{\infty} \frac{n^{p+1}}{(p+1)^n}.$$

Here, to save space, omit the tables of values, and just use graphical displays as in part (a)(iii). Click on the graphs to estimate the values of the sums for the series. Also use the root test to determine, in general, for what values of  $p > 0$  the series converges (hand in this work too).

**Infinite Products:** The concept of taking the product of infinitely many numbers is a natural analog of forming the sum of infinitely many numbers. Both are ideal concepts and involve using limits to formulate precisely. The notation for an infinite product is, naturally enough,

$$\prod_{n=1}^{\infty} b_n = b_1 b_2 b_3 \cdots,$$

where  $\{b_n\}_{n=1}^{\infty}$  is a sequence of real numbers. Just as with infinite sums, we have to agree upon (or define) what we mean by such a product, since calculating manually, or by computer, infinitely many products is impossible in practice. Of course we can always take the product of *finitely* many terms, and thus the sequence of partial products

$$\begin{aligned} pr_1 &= b_1 \\ pr_2 &= b_1 b_2 \\ &\vdots \\ pr_n &= b_1 b_2 \cdots b_n \\ &\vdots \end{aligned}$$

is a well-defined sequence (and we can compute as many of the  $pr_n$ 's as we wish with a computer). Thus, we get a sequence  $\{pr_n\}_{n=1}^{\infty}$ , which is called