```
restart
with(Physics[Vectors]) :
with(VectorCalculus) :
Step(mathematicalnotation = true)
SetCoordinates('cartesian'[i, j, k]);
```

$$\mathit{Step}(\mathit{mathematicalnotation} = \mathit{true})\ \mathit{SetCoordinates}\left(\mathit{cartesian}_{i,\ j,\ k}\right) \tag{1}$$

```
UU(x, y, z) := ux(x, y, z)·_i + uy(x, y, z)·_j + uz(x, y, z)·_k;
```

$$UU := (x,\ y,\ z) \mapsto ux(x,\ y,\ z)\cdot\hat{i} + uy(x,\ y,\ z)\cdot\hat{j} + uz(x,\ y,\ z)\cdot\hat{k} \tag{2}$$

```
∇(pressure(x, y, z));
Laplacian(UU(x, y, z));
BB(x, y, z) := 0·_i + 0·_j + 10·_k
```

$$\left(\frac{\partial}{\partial x}\ pressure(x,\ y,\ z)\right)\ \hat{i} + \left(\frac{\partial}{\partial y}\ pressure(x,\ y,\ z)\right)\ \hat{j} + \left(\frac{\partial}{\partial z}\ pressure(x,\ y,\ z)\right)\ \hat{k}$$

$$\left(\frac{\partial^2}{\partial x^2}\ ux(x,\ y,\ z) + \frac{\partial^2}{\partial y^2}\ ux(x,\ y,\ z) + \frac{\partial^2}{\partial z^2}\ ux(x,\ y,\ z)\right)\ \hat{i} + \left(\frac{\partial^2}{\partial x^2}\ uy(x,\ y,\ z) + \frac{\partial^2}{\partial y^2}\ uy(x,\ y,\ z) + \frac{\partial^2}{\partial z^2}\ uy(x,\ y,\ z)\right)\ \hat{j} + \left(\frac{\partial^2}{\partial x^2}\ uz(x,\ y,\ z) + \frac{\partial^2}{\partial y^2}\ uz(x,\ y,\ z) + \frac{\partial^2}{\partial z^2}\ uz(x,\ y,\ z)\right)\ \hat{k}$$

$$BB := (x,\ y,\ z) \mapsto 0 + 0 + 10\cdot\hat{k} \tag{3}$$

```
-∇(pressure(x, y, z)) + Laplacian(mu·UU(x, y, z)) + BB(x, y, z) = 0·_i + 0·_j + 0·_k;
```

$$\hat{i}\ \left(-\frac{\partial}{\partial x}\ pressure(x,\ y,\ z) + \mu\ \left(\frac{\partial^2}{\partial x^2}\ ux(x,\ y,\ z)\right) + \mu\ \left(\frac{\partial^2}{\partial y^2}\ ux(x,\ y,\ z)\right) + \mu\ \left(\frac{\partial^2}{\partial z^2}\ ux(x,\ y,\ z)\right)\right) + \hat{j}\ \left(-\frac{\partial}{\partial y}\ pressure(x,\ y,\ z) + \mu\ \left(\frac{\partial^2}{\partial x^2}\ uy(x,\ y,\ z)\right) + \mu\ \left(\frac{\partial^2}{\partial y^2}\ uy(x,\ y,\ z)\right) + \mu\ \left(\frac{\partial^2}{\partial z^2}\ uy(x,\ y,\ z)\right)\right) + \hat{k}\ \left(-\frac{\partial}{\partial z}\ pressure(x,\ y,\ z) + \mu\ \left(\frac{\partial^2}{\partial x^2}\ uz(x,\ y,\ z)\right) + \mu\ \left(\frac{\partial^2}{\partial y^2}\ uz(x,\ y,\ z)\right) + \mu\ \left(\frac{\partial^2}{\partial z^2}\ uz(x,\ y,\ z)\right) + 10\right) = 0 \tag{4}$$

```
Divergence(-∇(pressure(x, y, z)) + Laplacian(mu·UU(x, y, z)) + BB(x, y, z) = 0·_i + 0·_j
    + 0·_k);
```

$$-\frac{\partial^2}{\partial x^2}\,pressure(x,\ y,\ z)+\mu\left(\frac{\partial^3}{\partial x^3}\,ux(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial x\partial y^2}\,ux(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial x\partial z^2}\,ux(x,\ y,\ z)\right)-\frac{\partial^2}{\partial y^2}\,pressure(x,\ y,\ z)+\mu\left(\frac{\partial^3}{\partial x^2\partial y}\,uy(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial y^3}\,uy(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial y\partial z^2}\,uy(x,\ y,\ z)\right)-\frac{\partial^2}{\partial z^2}\,pressure(x,\ y,\ z)+\mu\left(\frac{\partial^3}{\partial x^2\partial z}\,uz(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial y^2\partial z}\,uz(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial z^3}\,uz(x,\ y,\ z)\right)=0 \tag{5}$$

```
algsubs(Divergence(UU(x, y, z)) = 0, %)
```

$$-\frac{\partial^2}{\partial x^2}\,pressure(x,\ y,\ z)+\mu\left(-\frac{\partial^3}{\partial x^2\partial y}\,uy(x,\ y,\ z)-\frac{\partial^3}{\partial x^2\partial z}\,uz(x,\ y,\ z)\right)+\mu\left(-\frac{\partial^3}{\partial y^3}\,uy(x,\ y,\ z)-\frac{\partial^3}{\partial y^2\partial z}\,uz(x,\ y,\ z)\right)+\mu\left(-\frac{\partial^3}{\partial y\partial z^2}\,uy(x,\ y,\ z)-\frac{\partial^3}{\partial z^3}\,uz(x,\ y,\ z)\right)-\frac{\partial^2}{\partial y^2}\,pressure(x,\ y,\ z)+\mu\left(\frac{\partial^3}{\partial x^2\partial y}\,uy(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial y^3}\,uy(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial y\partial z^2}\,uy(x,\ y,\ z)\right)-\frac{\partial^2}{\partial z^2}\,pressure(x,\ y,\ z)+\mu\left(\frac{\partial^3}{\partial x^2\partial z}\,uz(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial y^2\partial z}\,uz(x,\ y,\ z)\right)+\mu\left(\frac{\partial^3}{\partial z^3}\,uz(x,\ y,\ z)\right)=0 \tag{6}$$

```
simplify(%)
```

$$-\frac{\partial^2}{\partial x^2}\,pressure(x,\ y,\ z)-\frac{\partial^2}{\partial y^2}\,pressure(x,\ y,\ z)-\frac{\partial^2}{\partial z^2}\,pressure(x,\ y,\ z)=0 \tag{7}$$

```
Laplacian(-∇(pressure(x, y, z)) + Laplacian(mu·UU(x, y, z)) + BB(x, y, z) = 0·_i + 0·_j
    + 0·_k);
```

$$\left(\mu\left(\frac{\partial^4}{\partial z^4}\,ux(x,\ y,\ z)\right)+\mu\left(\frac{\partial^4}{\partial x^4}\,ux(x,\ y,\ z)\right)+2\ \mu\left(\frac{\partial^4}{\partial x^2\partial y^2}\,ux(x,\ y,\ z)\right)+2\ \mu\left(\frac{\partial^4}{\partial x^2\partial z^2}\,ux(x,\ y,\ z)\right)+\mu\left(\frac{\partial^4}{\partial y^4}\,ux(x,\ y,\ z)\right)+2\ \mu\left(\frac{\partial^4}{\partial y^2\partial z^2}\,ux(x,\ y,\ z)\right)-\frac{\partial^3}{\partial x\partial z^2}\,pressure(x,\ y,\ z)-\frac{\partial^3}{\partial x^3}\,pressure(x,\ y,\ z)-\frac{\partial^3}{\partial x\partial y^2}\,pressure(x,\ y,\ z)\right)\hat{i}+\left(\mu\left(\frac{\partial^4}{\partial z^4}\,uy(x,\ y,\ z)\right)+\mu\left(\frac{\partial^4}{\partial x^4}\,uy(x,\ y,\ z)\right)+2\ \mu\left(\frac{\partial^4}{\partial x^2\partial y^2}\,uy(x,\ y,\ z)\right)+2\ \mu\left(\frac{\partial^4}{\partial x^2\partial z^2}\,uy(x,\ y,\ z)\right)+\mu\left(\frac{\partial^4}{\partial y^4}\,uy(x,\ y,\ z)\right)+2\ \mu\left(\frac{\partial^4}{\partial y^2\partial z^2}\,uy(x,\ y,\ z)\right)-\frac{\partial^3}{\partial y\partial z^2}\,pressure(x,\ y,\ z)-\frac{\partial^3}{\partial x^2\partial y}\,pressure(x,\ y,\ z)-\frac{\partial^3}{\partial y^3}\,pressure(x,\ y,\ z)\right) \tag{8}$$

$$\hat{j} + \left(2\ \mu\ \left(\frac{\partial^4}{\partial y^2 \partial z^2}\ uz(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial z^4}\ uz(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial x^4}\ uz(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial y^2}\ uz(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial z^2}\ uz(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial y^4}\ uz(x,\ y,\ z) \right) - \frac{\partial^3}{\partial z^3}\ pressure(x,\ y,\ z) - \frac{\partial^3}{\partial x^2 \partial z}\ pressure(x,\ y,\ z) - \frac{\partial^3}{\partial y^2 \partial z}\ pressure(x,\ y,\ z) \right) \hat{k} = 0$$

```
algsubs( - ∂^2/∂x^2 pressure(x, y, z) - ∂^2/∂y^2 pressure(x, y, z) - ∂^2/∂z^2 pressure(x, y, z) = 0,
    % );
```

$$\left(\mu\ \left(\frac{\partial^4}{\partial z^4}\ ux(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial x^4}\ ux(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial y^2}\ ux(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial z^2}\ ux(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial y^4}\ ux(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial y^2 \partial z^2}\ ux(x,\ y,\ z) \right) \right) \hat{i} + \left(\mu\ \left(\frac{\partial^4}{\partial z^4}\ uy(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial x^4}\ uy(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial y^2}\ uy(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial z^2}\ uy(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial y^4}\ uy(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial y^2 \partial z^2}\ uy(x,\ y,\ z) \right) \right) \hat{j} + \left(2\ \mu\ \left(\frac{\partial^4}{\partial y^2 \partial z^2}\ uz(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial z^4}\ uz(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial x^4}\ uz(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial y^2}\ uz(x,\ y,\ z) \right) + 2\ \mu\ \left(\frac{\partial^4}{\partial x^2 \partial z^2}\ uz(x,\ y,\ z) \right) + \mu\ \left(\frac{\partial^4}{\partial y^4}\ uz(x,\ y,\ z) \right) \right) \hat{k} = 0 \tag{9}$$

```
collect(%, mu);
```

$$\left(\left(\frac{\partial^4}{\partial z^4}\ ux(x,\ y,\ z) + \frac{\partial^4}{\partial x^4}\ ux(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial x^2 \partial y^2}\ ux(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial x^2 \partial z^2}\ ux(x,\ y,\ z) + \frac{\partial^4}{\partial y^4}\ ux(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial y^2 \partial z^2}\ ux(x,\ y,\ z) \right) \hat{i} + \left(\frac{\partial^4}{\partial z^4}\ uy(x,\ y,\ z) + \frac{\partial^4}{\partial x^4}\ uy(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial x^2 \partial y^2}\ uy(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial x^2 \partial z^2}\ uy(x,\ y,\ z) + \frac{\partial^4}{\partial y^4}\ uy(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial y^2 \partial z^2}\ uy(x,\ y,\ z) \right) \hat{j} + \left(2\ \frac{\partial^4}{\partial y^2 \partial z^2}\ uz(x,\ y,\ z) + \frac{\partial^4}{\partial z^4}\ uz(x,\ y,\ z) + \frac{\partial^4}{\partial x^4} \right. \tag{10}$$

$$uz(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial x^2 \partial y^2}\ uz(x,\ y,\ z) + 2\ \frac{\partial^4}{\partial x^2 \partial z^2}\ uz(x,\ y,\ z) + \frac{\partial^4}{\partial y^4}\ uz(x,\ y,\ z)\Bigg)$$

$$\hat{k}\Bigg)\ \mu = 0$$