7

see opgave 2

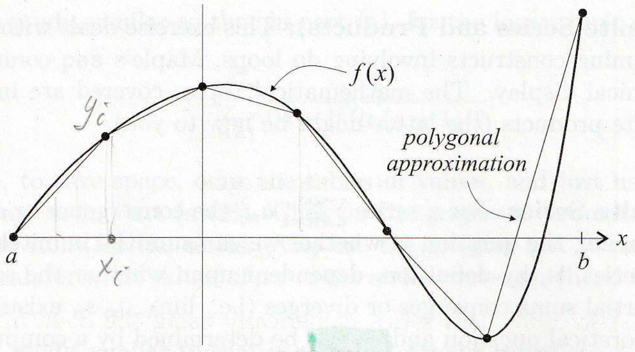

Figure 3.11: *A polygonal approximation to the curve C which is the graph of f on the interval* $[a, b]$.

(b) Have f, a, b, and N as input to your procedure and the approximating length $AL(f)$ as output. Also have your procedure plot the approximating polygon and the graph of f in the same picture.

(c) Test your procedure with $f(x) = 4x/3$ on $[0, 3]$, for which you know the exact length. Does your procedure give the exact answer for any choice of N? Why?

(d) Use your procedure to approximate the length of the curve that is the graph of $f(x) = x^{\cos x}$ for $x \in [1, 15]$. Use $N = 10, 20, 50, 100$ and any other values you feel are appropriate. How large does N have to be before the graphs of f and the approximating polygon with N sides are indiscernible?

(e) Do (d) again but now with $f(x) = x^3/6 + 1/(2x)$ on the interval $[0.1, 2]$.

(f) Show that Equation (3.3) can be written as

$$d(Q_{i-1}, Q_i) = \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i \tag{3.4}$$

and use this to argue that the exact lengths of the curve which is the graph of f on $[a, b]$ is:

$$L(f) = \int_a^b \sqrt{1 + (f'(x))^2}\,dx. \tag{3.5}$$

(g) Use Formula (3.5) to find the exact lengths of the curves that are the graphs of the functions in parts (d) and (e). For (d) you will have to use Maple (so the answer is only a good approximation), but for (e) do the work by hand (turn in your work).