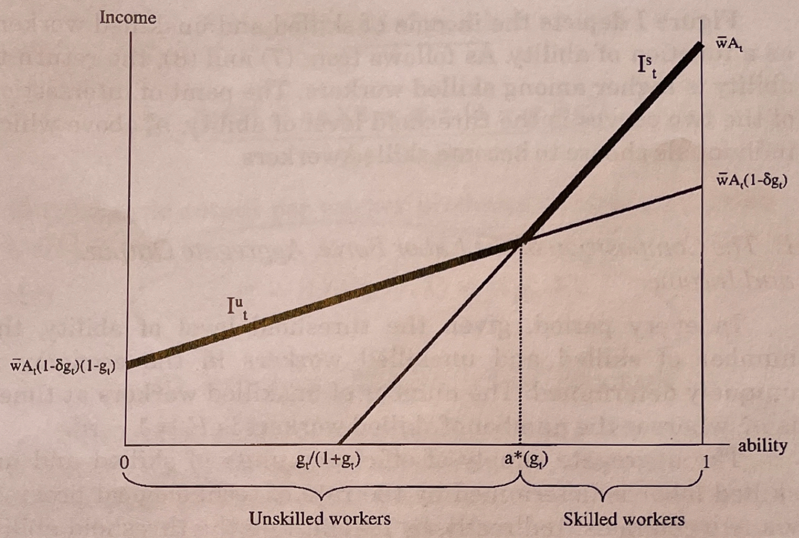

FIGURE I
The Threshold Level of Ability in Period t, a_t^*, and the Composition of the Labor Force, Given a Rate of Technological Progress g_t

both variables. Since individuals work only in the first period of their lives, maximization of first-period income is a necessary condition for maximization of utility. Individuals choose to become skilled workers if and only if, given their ability and the rate of technological progress, their income as skilled workers is higher than that as unskilled workers. That is, a member i of generation t whose ability level is a_t^i chooses to become a skilled worker if and only if $I^s(a_t^i, g_t, A_t) \geq I^u(a_t^i, g_t, A_t)$.

As follows from (7), (8), and (A1), there exists a unique, interior, threshold level of ability $0 < a_t^* < 1$. Individuals whose ability level is above this threshold would choose to become skilled workers, whereas individuals whose ability level is below it would choose to become unskilled workers. That is, $I^s(a_t^*, g_t, A_t) = I^u(a_t^*, g_t, A_t)$:

$$a_t^* = \frac{1 - \delta g_t + \delta g_t^2}{1 + \delta g_t^2} \equiv a^*(g_t), \qquad (9)$$

where the steady-state threshold level of ability decreases monotonically in the rate of technological progress, i.e., $a^*(g_t) < 0$, but is unaffected by the *level* of technology.

Suppose that the number of efficiency units of labor that a member i of generation t supplies as an unskilled worker, l_t^i, has a simple linear representation:[19]

$$l_t^i = l(a_t^i, g_t) = 1 - (1 - a_t^i)g_t. \quad (5)$$

The number of efficiency units of unskilled workers is subject to depreciation due to technological progress, and ability lessens the adverse effect of technological change. $[(1 - a_t^i)g_t]$ is therefore the fraction of the working time of an unskilled worker devoted to learning the new technology on the job. Individuals who choose to become unskilled workers are endowed with one efficiency unit of labor, regardless of ability. That is, the level of ability of unskilled workers would not be rewarded in a stationary technological environment.

Suppose that the number of efficiency units of labor that a member i of generation t supplies as a skilled worker has a simple linear representation:[20]

$$h_t^i = h(a_t^i, g_t) = (1 - \tau)[a_t^i - (1 - a_t^i)g_t]. \quad (6)$$

Individual i's level of human capital depends therefore on two components: a_t^i captures the direct positive effect of ability, whereas $[(1 - a_t^i)g_t]$ captures the depreciation due to technological progress.[21] In particular, unlike unskilled workers, the level of ability of skilled workers is rewarded even in a stationary technological environment.

Consistent with Schultz's observations, technological progress complements ability in the formation of human capital. That is, ability lessens the adverse effect of technological change. The higher the rate of technological progress the higher the return to ability. Individuals are subjected to two opposing effects due to technological progress. On the one hand, the potential number of efficiency units of labor is diminished due to the transition from the existing technological state to a superior one—the erosion

19. Assumption A1 (i.e., $0 < g_t < 1$) assures that regardless of ability, the number of efficiency units of any individual as an unskilled worker is strictly positive.

20. As will become apparent, in equilibrium individuals whose ability level would not assure a positive level of efficiency units as skilled workers would choose to become unskilled workers.

21. $[(1 - a_t^i)g_t]/a_t^i$ is the fraction of working time devoted for learning the new technology on the job. As one would expect, this formulation implies that, for a given level of ability, the time cost of unskilled workers is smaller than that of skilled workers.

effect. On the other hand, each individual operates with a superior level of technology—the productivity effect. Moreover, once the *rate* of technological progress reaches a steady state, the erosion effect is constant, whereas the productivity effect and thus wages grow at a constant rate. For the economy as a whole, the productivity effect is dominating, and producers would find it beneficial to adopt the new technology. However, for some individuals the erosion effect may dominate, and their wages would decline due to technological change.[22]

Occupational Choice. Since skilled and unskilled workers coexist in the labor market, additional structure is added to the model so as to assure that in equilibrium the number of skilled and unskilled workers is positive. In particular, the time cost of education τ is required to be compensated by β—the added weight given to skilled workers in the production technology so as to assure that regardless of the rate of technological progress investment in human capital is profitable. Although $\beta(1 - \tau) \geq 1$ would be a sufficient (additional) condition for the existence of skilled workers in the economy, to simplify the exposition it is assumed that $\beta(1 - \tau) = 1$.[23]

A member i of generation t who chooses to become a skilled worker supplies h_t^i efficiency units of skilled labor. Given the wage rate per efficiency unit of skilled labor at time t, w_t^s, the individual's income, $I_t^{i,s}$, is therefore

$$(7) \qquad I_t^{i,s} = w_t^s h_t^i = \overline{w} A_t[a_t^i - (1 - a_t^i)g_t] \equiv I^s(a_t^i, g_t, A_t),$$

where as depicted in Figure I, $\partial I^s(a_t^i, g_t, A_t)/\partial a_t^i = \overline{w} A_t [1 + g_t] > 0$, i.e., $I_t^{i,s}$ is an increasing linear function of a_t^i.

Similarly, the income of a member i of generation t who chooses to become an unskilled worker is

$$(8) \qquad I_t^{i,u} = w_t^u l_t^i = (1 - \delta g_t) A_t \overline{w}[1 - (1 - a_t^i) g_t] \equiv I^u(a_t^i, g_t, A_t),$$

where, as depicted in Figure I, $\partial I^u(a_t^i, g_t, A_t)/\partial a_t^i = (1 - \delta g_t, A_t) \overline{w} g_t > 0$; i.e., $I_t^{i,u}$ is an increasing linear function of a_t^i.

As stated earlier, individuals' utility function is defined over first- and second-period consumption, and is strictly increasing in

22. The underlying assumption is that the new technology replaces the old technology. This outcome can be endogenized in several ways (e.g., intertemporal consideration of firms and cost of technology adoption that is convex in the distance of the technological frontier).

23. The existence of unskilled workers in the economy follows directly from (A1), (5), and (6), for any $\beta > 0$.