

num211  
input:

$$n = 2, s = 3$$

$$(\Rightarrow sn = 6)$$

$$M = \begin{pmatrix} 1 & 4 \\ 3 & 0 \end{pmatrix} \in \mathbb{R}^{n \times n}$$

$$g := t \mapsto e^{-2t} \begin{pmatrix} 4 \\ 3 \end{pmatrix} \in \mathbb{R}^n$$

$$f := (t, x) \mapsto M \cdot x + g(t) \in \mathbb{R}^n$$

$$x_0 := \begin{pmatrix} 3 \\ 5 \end{pmatrix} \in \mathbb{R}^n$$

$$A := \begin{pmatrix} 0 & 0 & 0 \\ 1/4 & 1/4 & 0 \\ 0 & 1 & 0 \end{pmatrix} \in \mathbb{R}^{s \times s}$$

$$b := \begin{pmatrix} 1/6 \\ 2/3 \\ 1/6 \end{pmatrix} \in \mathbb{R}^s$$

$$c := \begin{pmatrix} 0 \\ 1/2 \\ 1 \end{pmatrix} \in \mathbb{R}^s$$

$$h := 0.00001$$

(The code should also work for other numbers, this is just an example).

I want that the following system of equations gets solved (you can use the built in function for the identity matrix and the Kronecker-product from the LinearAlgebra package):

$$X = X_0 + h \cdot \left( \text{KroneckerProduct}(A, \text{IdentityMatrix}(n)) \cdot F(T, X) \right)$$

where

$$X := \begin{pmatrix} x_1 \\ \vdots \\ x_s \end{pmatrix} \in \mathbb{R}^{sn} \text{ (Each } x_i \text{ has dimension } n \text{ and we have } s \text{ of such } x_i, \text{ meaning } x_1 = (x_{1,1}, \dots, x_{1,n}))$$

$$X_0 := \begin{pmatrix} x_0 \\ \vdots \\ x_0 \end{pmatrix} \in \mathbb{R}^{sn} \text{ (s entries of the vector } x_0)$$

$$F(T, X) := \begin{pmatrix} f(hc_1, x_1) \\ \vdots \\ f(hc_s, x_s) \end{pmatrix} \in \mathbb{R}^{sn} \text{ (f maps into } \mathbb{R}^n \text{ and we have } s \text{ entries)}$$

So maple should solve the above system for the vectors  $x_1, \dots, x_s$  (each of them is in  $\mathbb{R}^n$ ) and then evaluate  
output:

$$x_0 + h(b_1 f(c_1 h, x_1) + \dots + b_s f(c_s h, x_s))$$

I am not 100% sure whether I did any mistakes but I ended up with the solution

$$\begin{pmatrix} 2.99899255509596 \\ 4.99919047401420 \end{pmatrix}$$

for the above example, if you want something to compare. But I was not able to code the above steps as a procedure, I did it step by step.