Solve

$$\ln\left(\cos\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right)\right)+\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right)\tan\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right)=y(x)$$

This is d'Alembert ODE. It has the form

$$y(x)=xf(y'(x)+g(y'(x)$$

Where f,g are functions of $y'(x)$. If x is missing, then f,g must both be nonlinear in p for the ODE to be d'Alembert ODE.

Solving for $y(x)$ in the differential equation gives the following

$$y(x)=\ln\left(\cos\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right)\right)+\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right)\tan\left(\frac{\mathrm{d}}{\mathrm{d}x}y(x)\right) \tag{1}$$

Replacing $\frac{\mathrm{d}}{\mathrm{d}x}y(x)$ by $p(x)$, the above becomes

$$\begin{aligned} y(x)&=f+g \\ f&=p\tan(p) \\ g&=\ln(\cos(p)) \end{aligned} \tag{1}$$

ODE (1) is now solved.

$$y(x)=p\tan(p)+\ln(\cos(p)) \tag{1}$$

Since

$$\frac{df}{dx}=\left(\frac{\mathrm{d}}{\mathrm{d}x}p(x)\right)\tan(p(x))+p(x)\left(\frac{\mathrm{d}}{\mathrm{d}x}p(x)\right)\left(1+(\tan(p(x)))^2\right)$$

And

$$\frac{dg}{dx}=-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}x}p(x)\right)\sin(p(x))}{\cos(p(x))}$$

Then taking derivatives of (1) w.r.t. x and remembering that $p(x)$ is a function of x gives

$$\begin{aligned} p&=\frac{df}{dx}+\frac{dg}{dx} \\ &=\left(\tan(p)+p\left(1+(\tan(p))^2\right)-\frac{\sin(p)}{\cos(p)}\right)\frac{\mathrm{d}p}{\mathrm{d}x} \end{aligned} \tag{2}$$

The singular solution is found when $\frac{\mathrm{d}p}{\mathrm{d}x}=0$. Solving the above for p gives

$$p=0$$

Substituting $p=0$ values in (1) gives the singular solution

$$\begin{aligned} y(x)&=\ln(\cos(0)) \\ &=0 \end{aligned}$$

The general solution is found when $\frac{dp}{dx} \neq 0$. From (2) this results in

$$\frac{dp}{dx} = p\left(\tan(p) + p\left(1 + (\tan(p))^2\right) - \frac{\sin(p)}{\cos(p)}\right)^{-1}$$

Inverting the above gives

$$\frac{dx}{dp} = \frac{1}{p}\left(\tan(p) + p\left(1 + (\tan(p))^2\right) - \frac{\sin(p)}{\cos(p)}\right)$$

$x(p)$ is now the dependent variable and p as the independent variable. Now this ODE is solved for $x(p)$. Solving for $\frac{d}{dp}x(p)$ in $\frac{d}{dp}x(p) - \frac{1}{p}\left(\tan(p) + p\left(1 + (\tan(p))^2\right) - \frac{\sin(p)}{\cos(p)}\right) = 0$ gives

$$\frac{d}{dp}x(p) = -\frac{-\cos(p)(\tan(p))^2 p - \tan(p)\cos(p) - p\cos(p) + \sin(p)}{p\cos(p)}$$

$x(p)$ is now found by integration. Hence

$$x(p) = \int -\frac{-\cos(p)(\tan(p))^2 p - \tan(p)\cos(p) - p\cos(p) + \sin(p)}{p\cos(p)}\, dp = \tan(p) + C_1$$

Solving for p from the above in terms of x gives

$$p = \arctan(x - C_1)$$

Substituting the above solution for p in Eq (1) gives the general solution.

$$y(x) = \arctan(x - C_1)\, x - \arctan(x - C_1)\, C_1 - \frac{\ln\left(1 + (x - C_1)^2\right)}{2}$$

Verification of solutions

$$\boxed{y(x) = 0}$$

Verified OK

$$\boxed{y(x) = \arctan(x - C_1)\, x - \arctan(x - C_1)\, C_1 - \frac{\ln\left(1 + (x - C_1)^2\right)}{2}}$$

Verified OK

To compare with Maple

$$y(x) = 0$$

$$x - \int^{y(x)} (\mathrm{RootOf}(\ln(\cos(\_Z)) + \_Z\tan(\_Z) - \_a))^{-1} d\_a - \_C1 = 0$$