We write the PDE in the index notation:

$$(\lambda+\mu)u_{j,ji}+\mu u_{i,jj}-\frac{1}{4}\mu l^2 u_{i,kkjj}=\rho\ddot{u}_i.$$

We multiply the equation by an arbitrary but time-independent vector field $v_i$

$$(\lambda+\mu)u_{j,ji}v_i+\mu u_{i,jj}v_i-\frac{1}{4}\mu l^2 u_{i,kkjj}v_i=\rho\ddot{u}_i v_i,$$

and then calculate the individual terms:

$$\begin{aligned}
u_{j,ji}v_i &= u_{i,ij}v_j=(u_{i,i}v_j)_{,j}-u_{i,i}v_{j,j},\\
u_{i,jj}v_i &= (u_{i,j}v_i)_{,j}-u_{i,j}v_{i,j},\\
u_{i,kkjj}v_i &= (u_{i,kkj}v_i)_{,j}-u_{i,kkj}v_{i,j}=(u_{i,kkj}v_i)_{,j}-(u_{i,kk}v_{i,j})_{,j}+u_{i,kk}v_{i,jj}.
\end{aligned}$$

Let $\Omega$ be the domain of the PDE and $\Gamma$ be its boundary, and let $n$ be the outward unit normal vector to $\Gamma$. We integrate the equation over $\Omega$ and apply the Divergence Theorem to get

$$\begin{aligned}
&\int_\Gamma\Big[(\lambda+\mu)u_{i,i}v_j+\mu u_{i,j}v_i-\frac{1}{4}\mu l^2(u_{i,kkj}v_i-u_{i,kk}v_{i,j})\Big]n_j\,dA\\
&\qquad-\int_\Omega\Big[(\lambda+\mu)u_{i,i}v_{j,j}+\mu u_{i,j}v_{i,j}-\frac{1}{4}\mu l^2 u_{i,kk}v_{i,jj}\Big]\,dV=\frac{d^2}{dt^2}\int_\Omega\rho u_i v_i\,dV.
\end{aligned}$$