```
> restart:
```

Pb 8
How to solve it by hand?
Here is the "classical" method, incomplete because we must account for periodic solutions too.
For instance cos(2*u)= ±(1-tan(u)^2)/(1-tan(u)^2);

I let you figure out how to find the second solution (your "true" one)

```
> eq := arccos((x^2-1)/(x^2+1)) + arctan(2*x/(x^2-1))=2*Pi/3
```

$$eq := \arccos\left(\frac{x^2-1}{x^2+1}\right) + \arctan\left(\frac{2\,x}{x^2-1}\right) = \frac{2}{3}\,\pi \quad (1$$

```
> # observation 1: a classical relation

  'tan(2*u)' = 2*tan(u)/(1-tan(u)^2);

  # setting x=tan(u) this relation gives

  'arctan(2*x/(x^2-1))' = 'arctan(-tan(2*u))';

  # and simplifies to

  'arctan(2*x/(x^2-1))' = arctan(-tan(2*u));
```

$$\tan(2\,u) = \frac{2\tan(u)}{1-\tan(u)^2}$$

$$\arctan\left(\frac{2\,x}{x^2-1}\right) = \arctan(-\tan(2\,u))$$

$$\arctan\left(\frac{2\,x}{x^2-1}\right) = -\arctan(\tan(2\,u)) \quad (2$$

```
> # observation 2: another classical relation
  # still setting x=tan(u)

  'cos(2*u)' = -(tan(u)^2-1)/(1-tan(u)^2);

  # which gives

  'arccos((x^2-1)/(x^2+1))' = 'arccos(-cos(2*u))';
```

```
# and simplifies to
'arccos((x^2-1)/(x^2+1))' = arccos(-cos(2*u));
```

$$\cos(2\,u) = -\frac{\tan(u)^2 - 1}{1 - \tan(u)^2}$$

$$\arccos\left(\frac{x^2-1}{x^2+1}\right) = \arccos(-\cos(2\,u))$$

$$\arccos\left(\frac{x^2-1}{x^2+1}\right) = \pi - \arccos(\cos(2\,u)) \qquad (3$$

```
> # the equations thus treduces to
  - arccos(cos(2*u)) - arctan(tan(2*u)) = 2*Pi/3 - Pi
  # or
  arccos(cos(2*u)) + arctan(tan(2*u)) = Pi/3;
```

```
> restart:
```

Pb 9

Easily solvable by hand.

```
> M := Matrix(3$2, [1, 2, 2, 2, 1, -2, a, 2, b])
```

$$M := \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \qquad (4$$

```
> # Given that a and b appear only on the third line of M, there is
  # no M need to compute  C = M . transpose(M)
  # It's enough to compute C[3, 3] and one of C[1, 3] or C[2, 3].
  # Once done solve C[1, 3]=0 (or C[2, 3]=0) with respect to a (or b
  # if you prefer), inject this solution within the equation C[3, 3]=0
  # and solve with respect to b (or a)

  # Maple's solution
```

```
{a = 2/5, b = -11/5}, {a = -2, b = -1}
```

$$\left\{a=\frac{2}{5}, b=-\frac{11}{5}\right\}, \{a=-2, b=-1\} \quad (5$$

```
> restart:
```

Pb 6

Method 1
You can use some combinations of lines or rows to set the matrix into a product
M = L.U of two triangular matrices L and U whose inverses are obvious to compute
Then M^(-1) = U^(-1).L^(-1).

Method 2
Another way is to compute the matrix C of cofactors of M (which will give you the determinant D of M with no extra amount of work) and set M^(-1) = (1/D).C

Method 3
And you can also solve 3 linear systems M . X = Y(i)
where X is the column vector (a, b, c) and Y(i) the column vector whose all elements are 0 but element i.
To do this transform M into an upper triangular matrix by comninations of rows and do the same on (i)

```
> M__0 := Matrix(3$2, [-1, 1, 2, 1, 2, 3, 3, 1, 1]);
  Y__0 := Matrix(3$2, (i,j) -> `if`(j=i, 1, 0));
```

$$M_0 := \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$Y_0 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6$$

```
> # Method 3
  # first step to the upper triangular matrix (the rest is yours)
  # case i = 1 only, the rest is yours)

  M__1 := <
             M__0[1],
             M__0[2] - (M__0[2, 1]/M__0[1, 1]) *~ M__0[1],
             M__0[3] - (M__0[3, 1]/M__0[1, 1]) *~ M__0[1]
           >;

  Y__1 := <
             Y__0[1],
             Y__0[2] - (M__0[2, 1]/M__0[1, 1]) *~ Y__0[1],
             Y__0[3] - (M__0[3, 1]/M__0[1, 1]) *~ Y__0[1]
           >
```

$$M_1 := \begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix}$$

$$Y_1 := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

(7

```
> # Iterate this process on the adhoc submatrix of M__1 until to
  # obtain an upper triangular matrix.
  # Let M__? this matrix.
  # Then Y has became some Matrix Y__?
  # Solve M__? . X(i) = Y__?(i) for i=1..3 (as M__? is upper triangular
  # this is extremely simple if you this in the good direction).
  # Then the matrix whose column vectors are X(I), X(2) and X(3) is the
  # inverse matrix of M__0
```

```
> M__2 := <
             M__1[1],
             M__1[2],
             M__1[3] - (M__1[3, 2]/M__1[2, 2]) *~ M__1[2]
           >;
  Y__2 := <
             Y__1[1],
             Y__1[2],
             Y__1[3] - (M__1[3, 2]/M__1[2, 2]) *~ Y__1[2]
```

>

$$M_2 := \begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$Y_2 := \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{5}{3} & -\frac{4}{3} & 1 \end{bmatrix} \quad (8$$

```
> # Solution
  X := Matrix(3, 3, [1, -1, 1, -8, 7, -5, 5, -4, 3])
```

$$X := \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \quad (9$$

```
> # check
  X . M__0   ,   M__0 . X
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10$$