soliton structures of dromions, peakons, and foldons? Motivated by these issues, we consider the following (2+1)-dimensional dispersive long wave equation (DLWE):

$$u_{ty}+v_{xx}+u_xu_y+uu_{xy}=0,$$
$$v_t+(uv)_x+u_{xxy}=0, \tag{1}$$

which was introduced by Boiti[14] as a compatibility for a weak lax pair. The (1+1)-dimensional DLWE ($y=x$ of Eq. (1)) is called the classical Boussinesq equation. There exist a large number of papers discussing the possible applications and the exact solutions of the (1+1)-dimensional DLWE.[15] In Ref. [16], Paquin and Winternitz showed that the symmetry algebra of Eq. (1) is infinite-dimensional and has the Kac–Moody–Virasoro structure. The variable separation solutions of Eq. (1) have been obtained by the general projective Riccati equation method.[17]

2. Variable separation solutions

In order to solve Eq. (1), first, let us make a transformation for Eq. (1): $v=u_y$. Substituting the transformation into Eq. (1) yields

$$u_{ty}+(u_xu)_y+u_{xxy}=0. \tag{2}$$

Next we suppose that Eq. (2) has the following formal solution:

$$u=a_0+a_1\phi(R)+a_2\sqrt{[A\phi(R)-a][B\phi(R)-b]}, \tag{3}$$

where a_0, a_1, a_2, and R are functions of $\{x,y,t\}$ to be determined. The ϕ satisfies a new project equation[18]

$$\mathrm{d}\phi/\mathrm{d}R=(A\phi-a)(B\phi-b), \tag{4}$$

with the general solution

$$\phi=\frac{b\exp[(aB-Ab)R]-a\exp[C_1(Ab-aB)]}{B\exp[(aB-Ab)R]-A\exp[C_1(Ab-aB)]}, \tag{5}$$

where C_1 is an integration constant, and A, B, a, and b are arbitrary constants. Inserting Eqs. (3) and (4) into Eq. (2), selecting the variable separation ansatz

$$R=p(x,t)+q(y), \tag{6}$$

and eliminating all the coefficients of the powers of ϕ^i and $\sqrt{(A\phi-a)(B\phi-b)}$, we obtain a set of partial differential equations, from which we derive two special solutions, namely,

$$a_0=[(Ab+Ba)p_x^2-p_t-p_{xx}]/p_x,$$
$$a_1=-2ABp_x,\quad a_2=0, \tag{7}$$

and

$$a_0=-(p_t+p_{xx})/p_x,\quad a_1=-ABp_x,$$
$$a_2=\pm\sqrt{AB}p_x,\quad Ab+Ba=0, \tag{8}$$

where $p\equiv p(x,t)$ and $q\equiv q(y)$.

Therefore, from Eqs. (3) and (5)–(8), we can derive the variable separation solutions of the (2+1)-dimensional DLWE. According to Eqs. (3), (5), (6), and (8), the physical quantity v reads

$$\begin{aligned}v&=u_y\\&=\left\{-\frac{p_t+p_{xx}}{p_x}\right.\\&\qquad\left.+abp_x\left[\frac{\Gamma_+}{\Gamma_-}\pm\sqrt{\left(\frac{\Gamma_+}{\Gamma_-}+1\right)\left(\frac{\Gamma_+}{\Gamma_-}-1\right)}\right]\right\}_y\\&=-2A^2b^2Bp_xq_y\exp[-2Ab(p+q)]\\&\qquad\times\left[\frac{1}{\Gamma_-}+\frac{\Gamma_+}{\Gamma_-^2}-\frac{4A\exp(2AbC_1)\Gamma_+}{\Gamma_-^3\sqrt{\left(\frac{\Gamma_+}{\Gamma_-}+1\right)\left(\frac{\Gamma_+}{\Gamma_-}-1\right)}}\right],\end{aligned} \tag{9}$$

where $\Gamma_\pm=A\exp(2AbC_1)\pm B\exp[-2Ab(p+q)]$, $p\equiv p(x,t)$, and $q\equiv q(y)$.

3. The non-completely elastic interactions

In this section, we will focus on the interaction between special multi-solitons for the physical quantity v expressed by Eq. (9). Firstly, we discuss three special multi-soliton structures, i.e., special multi-dromions, dromion-like multi-peakons, and dromion-like multi-semifoldons, by introducing the multi-valued function

$$p_x=0.5\mathrm{sech}^2(\zeta-0.5t),$$
$$x=\zeta-C\tanh(\zeta-0.5t), \tag{10}$$

with

$$q=\sum_{j=-N}^{N}b_j\tanh(l_jy+y_{0j}), \tag{11}$$

where C is a characteristic parameter that determines the localized structure. From Eq. (10), we can know that ζ is a multi-valued function in some suitable regions of x, and function p_x is a multi-valued function of x in these areas, though it is a single-valued function of ζ. From Eq. (11), we can construct $(2N+1)$ dromions. Here we choose $N=1, b_j=1, l_j=0.5$, and $y_{0j}=4j$.