

"Is there a convert command that converts a matrix to a tensor"

First of all, even working with matrices, you are working in some *space* and, in general, using some coordinates, and if working with tensors you use certain type of letters as *tensor indices*. Suppose it is a 3D Euclidean space, your coordinates are Cartesian, and you want to use lowercase letters as indices; set that:

The dimension and signature of the tensor space are set to [3,(+++)]

Systems of spacetime coordinates are : $\{X = (x, y, z)\}$

The Euclidean metric in coordinates $[x, y, z]$

$$g_{\mu,\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} &[coordinatesystems = \{X\}, dimension = 3, \\ &metric = \{(1, 1) = 1, (2, 2) = 1, (3, 3) = 1\}, \\ &spacetimeindices = lowercaselatin] \end{aligned} \tag{1}$$

Suppose you have the matrix

$$\begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{2}$$

To transform it into a tensor, just use the Define command

Defined objects with tensor properties

$$\{\gamma_a, M_{i,j}, \sigma_a, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \tag{3}$$

That is all. You can perform all tensor computations with such so defined tensor $M_{i,j}$. Start with its definition

$$M_{i,j} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{4}$$

Then its components (default is the *covariant* ones)

$$M_{a,b} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{5}$$

The *contravariant* components are the same as the covariant ones because the space is Euclidean

$$M^{a,b} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \quad (6)$$

The trace

$$m_{1,1} + m_{2,2} + m_{3,3} \quad (7)$$

Also via

$$M_{j,j} \quad (8)$$

$$m_{1,1} + m_{2,2} + m_{3,3} \quad (9)$$

The determinant

$$\begin{aligned} & m_{1,1}m_{2,2}m_{3,3} - m_{1,1}m_{2,3}m_{3,2} - m_{1,2}m_{2,1}m_{3,3} \\ & + m_{1,2}m_{2,3}m_{3,1} + m_{1,3}m_{2,1}m_{3,2} - m_{1,3}m_{2,2}m_{3,1} \end{aligned} \quad (10)$$

The inert representation of this determinant

$$| \text{M} | \quad (11)$$

$$\begin{aligned} & m_{1,1}m_{2,2}m_{3,3} - m_{1,1}m_{2,3}m_{3,2} - m_{1,2}m_{2,1}m_{3,3} \\ & + m_{1,2}m_{2,3}m_{3,1} + m_{1,3}m_{2,1}m_{3,2} - m_{1,3}m_{2,2}m_{3,1} \end{aligned} \quad (12)$$

The non-zero components

$$\begin{aligned} M_{a,b} = \{ & (1,1) = m_{1,1}, (1,2) = m_{1,2}, (1,3) = m_{1,3}, (2,1) = m_{2,1}, (2,2) = m_{2,2}, \\ & (2,3) = m_{2,3}, (3,1) = m_{3,1}, (3,2) = m_{3,2}, (3,3) = m_{3,3} \} \end{aligned} \quad (13)$$

*"for example to perform basic operations like $\text{omega} * J * \text{omega}$ where J is the inertia tensor and omega the vector of angular velocity?"*

OK. Define your *vector* now. You can do that directly with Define, or also starting with a Maple *Vector* constructor

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (14)$$

Defined objects with tensor properties

$$\{\gamma_a, M_{i,j}, \sigma_a, V_j, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \quad (15)$$

For example, the computation you asked for

$$V_i M_{i,j} V_j \quad (16)$$

$$m_{1,1}v_1^2 + m_{1,2}v_1v_2 + m_{1,3}v_1v_3 + m_{2,1}v_1v_2 + m_{2,2}v_2^2 + m_{2,3}v_2v_3 + m_{3,1}v_1v_3 + m_{3,2}v_2v_3 + m_{3,3}v_3^2 \quad (17)$$

Another example: operations with the metric

$$V_a g_{a,i} M_{i,j} g_{b,j} V_b \quad (18)$$

$$V_i M_{i,j} V_j \quad (19)$$

Or products of tensor (underlying matrices) that take the symmetry properties into account

$$\epsilon_{a,b,c} V_a V_b \quad (20)$$

$$0 \quad (21)$$

Or other tensor commands

$$\frac{M_{a,b}}{2} + \frac{M_{b,a}}{2} \quad (22)$$

$$\frac{M_{a,b}}{2} - \frac{M_{b,a}}{2} \quad (23)$$

Display components using Physics:-TensorArray

$$V_i M_{i,j} \quad (24)$$

$$\left[\begin{array}{ccc} m_{1,1}v_1 + m_{2,1}v_2 + m_{3,1}v_3 & m_{1,2}v_1 + m_{2,2}v_2 + m_{3,2}v_3 & m_{1,3}v_1 + m_{2,3}v_2 + m_{3,3}v_3 \end{array} \right] \quad (25)$$

$$V_k M_{i,j} \quad (26)$$

$$V_k M_{i,j} \text{ (ordering of free indices = [i,j,k])} \quad (27)$$

For all these and more check the large help page (sort-of-manual) (Physics:-Tensors

A note on the working space: *Physics* implements working with several different spaces simultaneously (space + spacetime +) - just use different kind of indices for each type of space, and there one kind of space that is *generic*, which means: *arbitrary-symbolic dimension* (all the other spaces have specific integer

dimension). The metric of such a *generic* space is Euclidean and represented by Physics:-KroneckerDelta

$$[genericindices = greek] \tag{28}$$

Defined objects with tensor properties

$$\{A_\mu, B_\mu, C_{\mu,\nu}, \gamma_a, M_{i,j}, \sigma_a, U_{a,b,c}, V_j, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \tag{29}$$

$$C_{\mu,\nu} - C_{\nu,\mu} \tag{30}$$

$$0 \tag{31}$$