

"Is there a convert command that converts a matrix to a tensor"

with(Physics):

First of all, even working with matrices, you are working in some *space* and, in general, using some coordinates, and if working with tensors you use certain type of letters as *tensor indices*. Suppose it is a 3D Euclidean space, your coordinates are Cartesian, and you want to use lowercase letters as indices; set that:

Setup(dimension = 3, metric = euclidean, coordinates = cartesian, spacetimeindices = lowercase)

The dimension and signature of the tensor space are set to [3,(+++)]

Systems of spacetime coordinates are : {X = (x, y, z)}

The Euclidean metric in coordinates [x, y, z]

$$g_{\mu,\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} &[coordinatesystems = \{X\}, dimension = 3, \\ &metric = \{(1, 1) = 1, (2, 2) = 1, (3, 3) = 1\}, \\ &spacetimeindices = lowercaselatin] \end{aligned} \tag{1}$$

Suppose you have the matrix

Matrix(3, symbol = m)

$$\begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{2}$$

To transform it into a tensor, just use the Define command

Define(M[i, j] = (2))

Defined objects with tensor properties

$$\{\gamma_a, M_{i,j}, \sigma_a, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \tag{3}$$

That is all. You can perform all tensor computations with such so defined tensor

$M_{i,j}$. Start with its definition

M[definition]

$$M_{i,j} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \tag{4}$$

Then itscomponents (default is the *covariant* ones)

$M[]$

$$M_{a,b} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \quad (5)$$

The *contravariant* components are the same as the covariant ones because the space is Euclidean

$M[]$

$$M^{a,b} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \quad (6)$$

The trace

$M[trace]$

$$m_{1,1} + m_{2,2} + m_{3,3} \quad (7)$$

Also via

$M[j,j]$

$$M_{j,j} \quad (8)$$

$SumOverRepeatedIndices((8))$

$$m_{1,1} + m_{2,2} + m_{3,3} \quad (9)$$

The determinant

$M[determinant]$

$$\begin{aligned} & m_{1,1}m_{2,2}m_{3,3} - m_{1,1}m_{2,3}m_{3,2} - m_{1,2}m_{2,1}m_{3,3} \\ & + m_{1,2}m_{2,3}m_{3,1} + m_{1,3}m_{2,1}m_{3,2} - m_{1,3}m_{2,2}m_{3,1} \end{aligned} \quad (10)$$

The inert representation of this determinant

$\%M[determinant]$

$$| \mathbf{M} | \quad (11)$$

$value((11))$

$$\begin{aligned} & m_{1,1}m_{2,2}m_{3,3} - m_{1,1}m_{2,3}m_{3,2} - m_{1,2}m_{2,1}m_{3,3} \\ & + m_{1,2}m_{2,3}m_{3,1} + m_{1,3}m_{2,1}m_{3,2} - m_{1,3}m_{2,2}m_{3,1} \end{aligned} \quad (12)$$

The non-zero components

$M[nonzero]$

$$M_{a,b} = \{(1,1) = m_{1,1}, (1,2) = m_{1,2}, (1,3) = m_{1,3}, (2,1) = m_{2,1}, (2,2) = m_{2,2}, (2,3) = m_{2,3}, (3,1) = m_{3,1}, (3,2) = m_{3,2}, (3,3) = m_{3,3}\} \quad (13)$$

"for example to perform basic operations like $\omega * J * \omega$ where J is the inertia tensor and ω the vector of angular velocity?"

OK. Define your *vector* now. You can do that directly with Define, or also starting with a Maple *Vector* constructor

$$\text{Vector}(3, \text{symbol} = v) \quad \left[\begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \right] \quad (14)$$

Define($V[j] = \mathbf{(14)}$)

Defined objects with tensor properties

$$\{\gamma_a, M_{i,j}, \sigma_a, V_j, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \quad (15)$$

For example, the computation you asked for

$$V[i]M[i,j]V[j] \quad V_i M_{i,j} V_j \quad (16)$$

SumOverRepeatedIndices($\mathbf{(16)}$)

$$m_{1,1}v_1^2 + m_{1,2}v_1v_2 + m_{1,3}v_1v_3 + m_{2,1}v_1v_2 + m_{2,2}v_2^2 + m_{2,3}v_2v_3 + m_{3,1}v_1v_3 + m_{3,2}v_2v_3 + m_{3,3}v_3^2 \quad (17)$$

Another example: operations with the metric

$$V[a]g_{-}[a,i]M[i,j]g_{-}[j,b]V[b] \quad V_a g_{a,i} M_{i,j} g_{b,j} V_b \quad (18)$$

Simplify($\mathbf{(18)}$)

$$V_i M_{i,j} V_j \quad (19)$$

Or products of tensor (underlying matrices) that take the symmetry properties into account

$$\text{LeviCivita}[a,b,c]V[a]V[b] \quad \epsilon_{a,b,c} V_a V_b \quad (20)$$

Simplify($\mathbf{(20)}$)

$$0 \quad (21)$$

Or other tensor commands

$$\text{Symmetrize}(M[a,b]) \quad \frac{M_{a,b}}{2} + \frac{M_{b,a}}{2} \quad (22)$$

$$\text{Antisymmetrize}(M[a,b]) \quad \frac{M_{a,b}}{2} - \frac{M_{b,a}}{2} \quad (23)$$

Display components using Physics:-TensorArray

$$V[i]M[i,j] \quad V_i M_{i,j} \quad (24)$$

$$\begin{aligned}
&TensorArray((\mathbf{24})) \\
&\left[\begin{array}{ccc} m_{1,1}v_1 + m_{2,1}v_2 + m_{3,1}v_3 & m_{1,2}v_1 + m_{2,2}v_2 + m_{3,2}v_3 & m_{1,3}v_1 + m_{2,3}v_2 + m_{3,3}v_3 \end{array} \right] \\
&V[k]M[i,j] \\
&V_k M_{i,j}
\end{aligned} \tag{26}$$

$$\begin{aligned}
&TensorArray((\mathbf{26}), explore) \\
&V_k M_{i,j} \text{ (ordering of free indices = [i, j, k])}
\end{aligned} \tag{27}$$

For all these and more check the large help page (sort-of-manual) (Physics:-Tensors

A note on the working space: *Physics* implements working with several different spaces simultaneously (space + spacetime +) - just use different kind of indices for each type of space, and there one kind of space that is *generic*, which means: *arbitrary-symbolic dimension* (all the other spaces have specific integer dimension). The metric of such a *generic* space is Euclidean and represented by Physics:-KroneckerDelta
Setup(genericindices = greek)

$$[genericindices = greek] \tag{28}$$

$$Define(A[mu], B[mu], C[mu, nu], symmetric)$$

Defined objects with tensor properties

$$\{A_\mu, B_\mu, C_{\mu,\nu}, \gamma_a, M_{i,j}, \sigma_a, U_{a,b,c}, V_j, \partial_a, g_{a,b}, \epsilon_{a,b,c}, X_a\} \tag{29}$$

$$\begin{aligned}
&C[mu, nu] - C[nu, mu] \\
&C_{\mu,\nu} - C_{\nu,\mu}
\end{aligned} \tag{30}$$

$$\begin{aligned}
&Simplify((\mathbf{30})) \\
&0
\end{aligned} \tag{31}$$