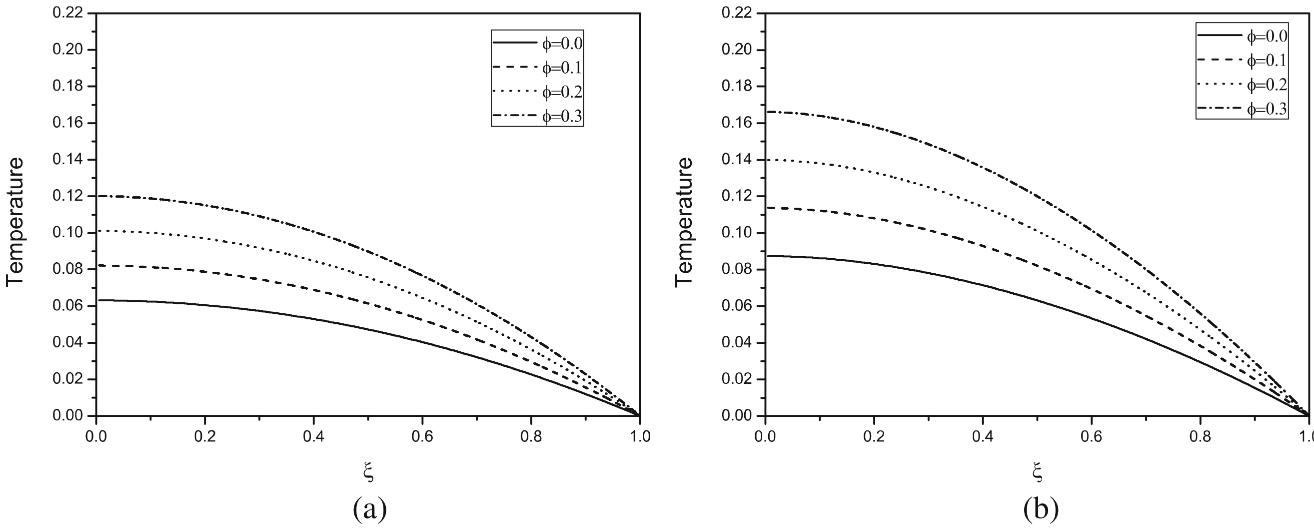

Fig. 3 Influence of ϕ on temperature in **a** parent **b** daughter arteries for fixed values of $s = 0.2$, $\beta = \pi/10$, and $G_r = 0.3$

and

$$(\lambda_d)_i \quad = \left| \frac{(z_{max} - z_3)\dfrac{dp}{dz}}{Q_d} \right| \quad \text{for} \quad z \geq z_3 \tag{24}$$

The shearing stress along both inner and outer walls of the bifurcated artery is determined by using

$$\tau = \frac{\mu_f}{(1-\phi)^{2.5}} \; \frac{1}{R} \; \frac{\partial w}{\partial \xi}. \tag{25}$$

4 Results and Discussions

The special case of nanofluid flow through a straight circular pipe can be obtained by taking $R_2(z) = 0$, $r_0 = 0$, $Gr = 0$, $\beta = 0$. The closed form expressions for the velocity and temperature field for the nanofluid flow through a circular pipe are given by ([23–25])

$$w(r) = -\frac{(1-\phi)^{2.5}}{4} \frac{\partial p}{\partial z} \left(1 - r^2\right) \quad \text{and} \tag{26}$$

$$\theta(r) = \frac{1}{4} [1 + 3\phi] s \left(1 - r^2\right)$$

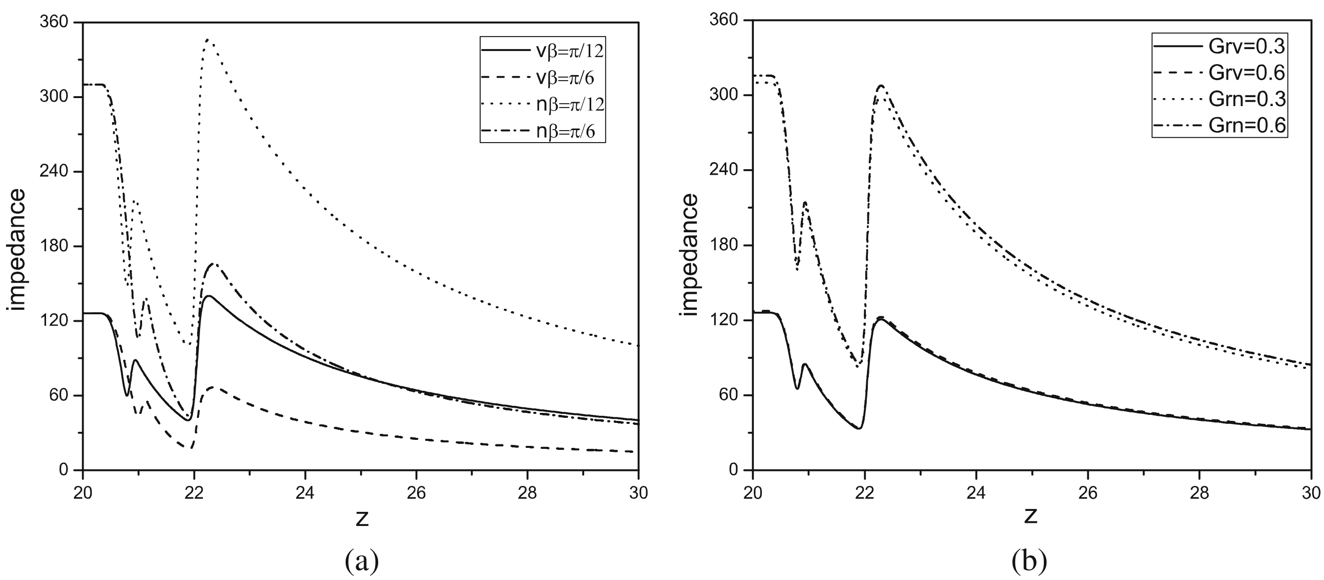

Fig. 4 Influence of **a** β for fixed values of $s = 0.2$, $G_r = 0.3$, and **b** G_r for fixed values of $s = 0.2$ and $\beta = \pi/10$ on impedance with and without nanoparticles on both sides of the apex

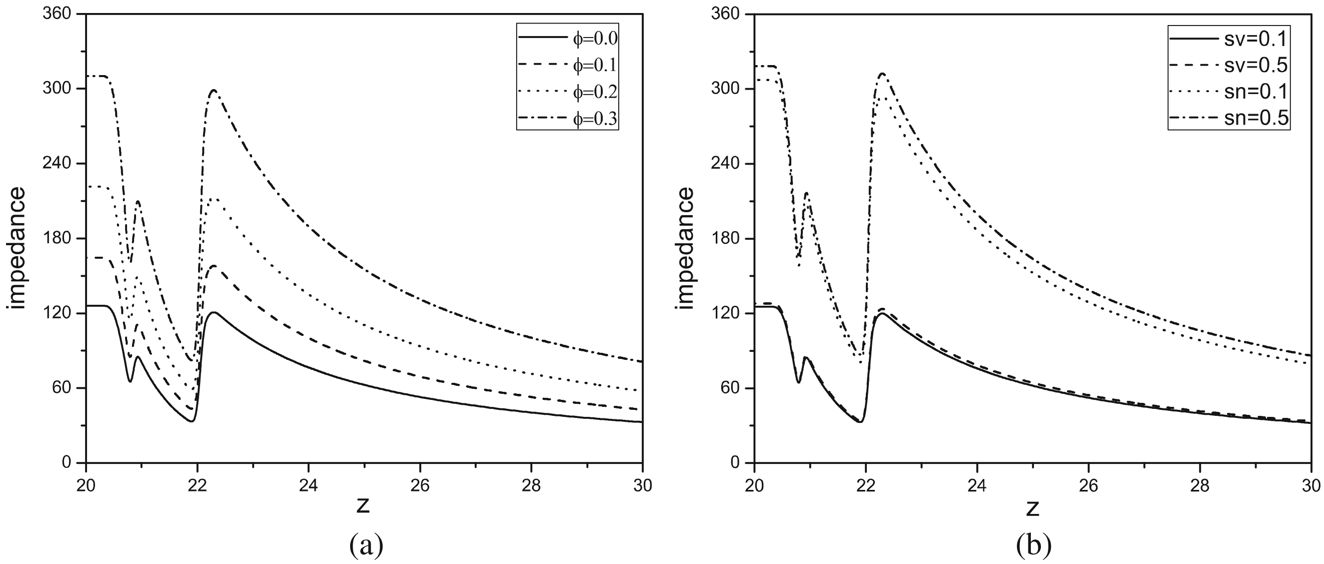

Fig. 5 Influence of **a** ϕ on impedance on both sides of the apex for fixed values of $\beta=\pi/10$, $s=0.2$, and **b** s on impedance in both sides of the apex with and without nanoparticles for fixed values of $\beta=\pi/10$ and $G_r=0.3$

In order to validate the accuracy of the present results, we have compared the results of temperature in the absence of $R_2(z)$, r_0, G_r, and β with the analytical solution by taking $s=0.2$ and $dp/dz = 2$. The comparison in the above case is found to be in good agreement, as shown in Table 1.

The aim of the present article is to examine the graphical features of pertinent parameters on the flow characteristics of blood through the stenosed bifurcated artery with blood as copper nanofluid. For better understanding of the analysis, the following data is used from [21]. $a = 5$ mm, $d' = 10$ mm, $l_0 = 5$ mm, $\beta = \frac{\pi}{10}$, $r_1 = 0.51a$, $\epsilon = 2a$, $s = 0.2$, $G_r = 0.3$, $\mu_f = 0.894$. In Fig. 2 to Fig. 10, the notation "−n" stands for fluid with nanoparicle volume fraction and "−v" stands for fluid without nanoparicles for all the variables.

The effect of β on temperature with and without nanoparticles both in the parent and the daughter arteries is presented in Fig. 2a, b. From these figures, it is observed that the temperature is increasing with the increase in the value of β both in the parent and the daughter arteries. Figure 3a, b shows the influence of nanoparticles on temperature both in the parent and daughter arteries. From these figures, it is noticed that the temperature is increasing with a rise in the value of ϕ. The temperature is enhanced with an enhancement in the value of s both in the parent and daughter arteries. It is very interesting to

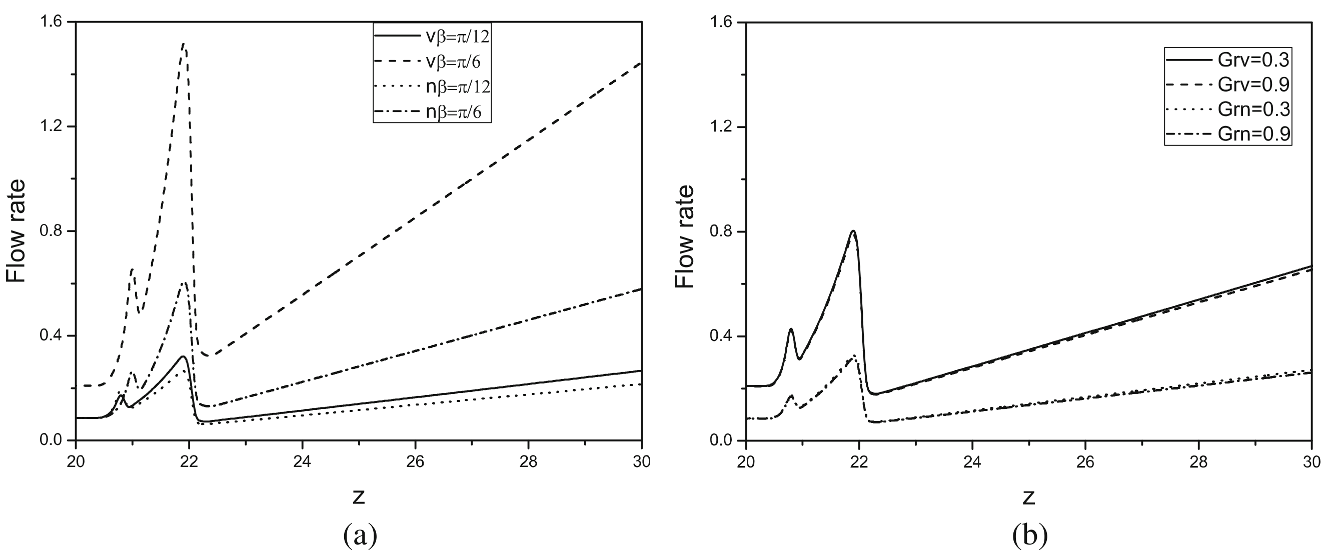

Fig. 6 Influence of **a** β for fixed values of $s=0.2$, $G_r=0.3$, and **b** G_r for fixed values of $s=0.2$ and $\beta=\pi/10$ on flow rate with and without nanoparticles on both sides of the apex

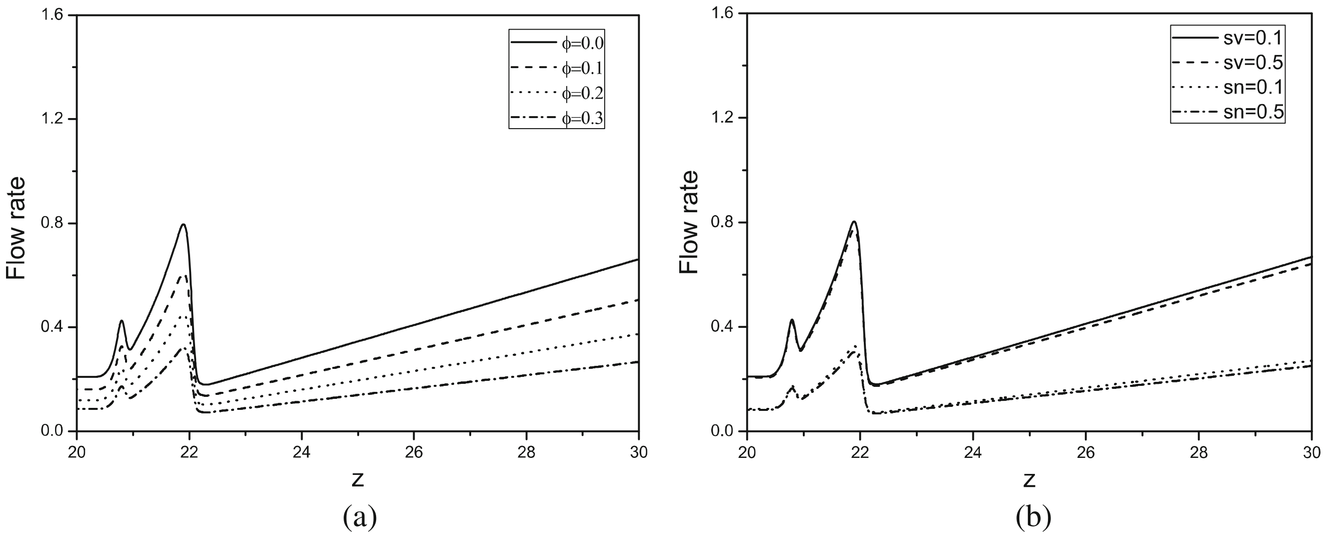

Fig. 7 Influence of **a** ϕ on flow rate on both sides of the apex for fixed values of $\beta = \pi/10$, $s = 0.2$, and **b** s on flow rate in both sides of the apex with and without nanoparticles for fixed values of $\beta = \pi/10$ and $G_r = 0.3$

note that temperature increases in case of pure water as compared to Cu–water because copper makes the arteries more flexible that slows down the velocity of blood flow and increase the temperature.

The variations of impedance on both sides of the apex with β and G_r with and without considering nanoparticles are plotted in Fig. 4a, b. From Fig. 4a, it is seen that the impedance is increasing with an increase in the value of β and decrease with nanoparticles on both sides of the apex. From Fig. 4b, it is noticed that impedance is decreasing with an increase in the value of G_r and nanoparticles on both sides of the apex, but the effect of G_r on impedance is insignificant. Figure 5a, b illustrates the effect of nanoparticle volume fraction (ϕ) and s on impedance on both sides of the flow divider. It is observed from Fig. 5a that the impedance is diminishing with an advancement in the value of ϕ on both sides of the flow divider. Figure 5b reveals that impedance is diminishing with an increase in the value of s and ϕ, but the effect of s on impedance is almost negligible. From Figs. 4 and 5, it is identified that impedance is increasing with an increase in the value of z, until inset of lateral junction, then a slight decrease occurred suddenly, and after that a gradual increase till the apex, and then a sudden decrease is identified. This is because of the divergence of the blood flow at the bifurcation of the artery. Thereafter, it is found that the impedance is uniform till z_{max}. It is noticed that the impedance increases in the case of pure water as compared to

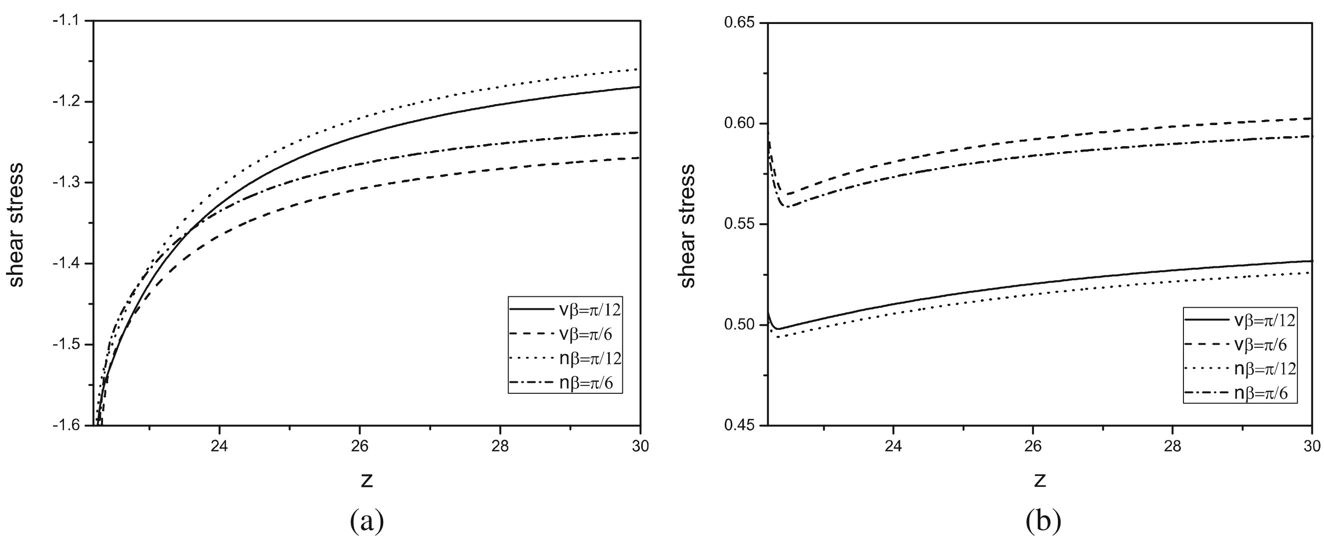

Fig. 8 Influence of β on shear stress along the **a** inner and **b** outer walls of the daughter artery with and without nanoparticles for fixed values of $G_r = 0.3$ and $s = 0.2$

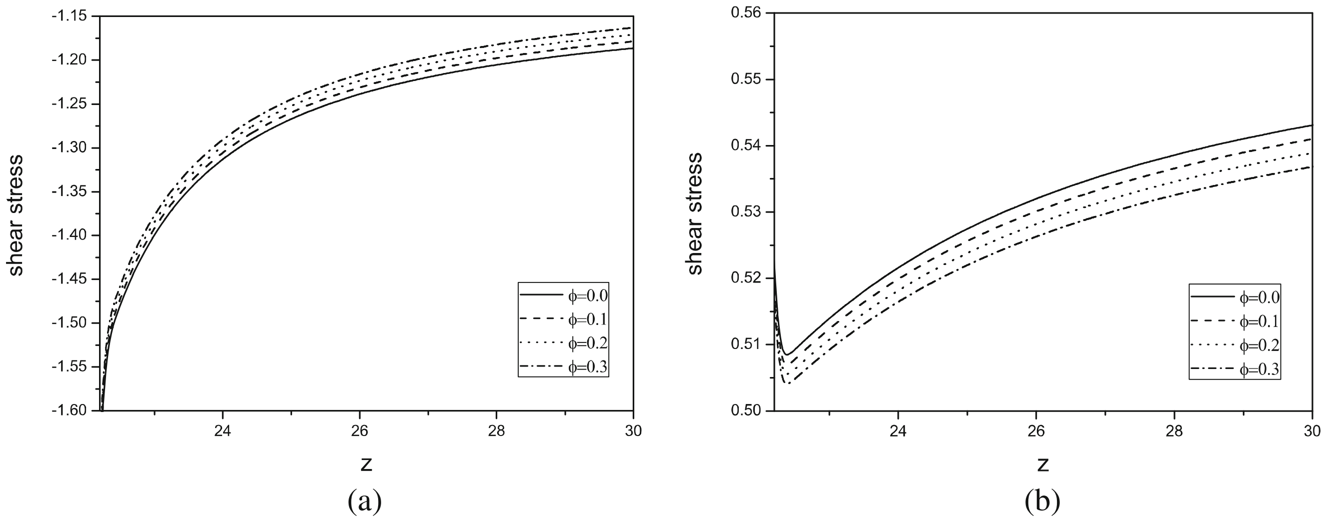

Fig. 9 Influence of ϕ on shear stress along the **a** inner and **b** outer walls of the daughter artery for fixed values of $G_r = 0.3$, $s = 0.2$, and $\beta = \pi/10$

copper–water because the density of the copper–water increases and it opposes the blood flow and decreases the impedance.

Figure 6a, depicts the influence of β and G_r on volumetric flow rate with and without nanoparticle on both sides of the apex. From Fig. 6a, it is seen that flow rate is increasing with an increase in the value of β and decreasing with the value of nanoparticles on both sides of the apex. From Fig. 6b, it is noticed that flow rate is decreasing with an increase in the value of G_r and ϕ, but the effect of G_r is almost negligible on flow rate. The influence of ϕ and s on flow rate near the apex is illustrated in Fig. 7a, b. It is observed from Fig. 7a that flow rate is diminishing with an increase in the value of ϕ on both sides of the apex. From Fig. 7b, it is depicted that flow rate is diminishing with enhanced values of s and ϕ on both sides of the apex. From Figs. 6a–7b, it is important to note that flow rate patterns are perturbed largely near the apex in the parent artery due to the presence of backflow at the start of the apex. All the profiles are locally increasing till inset of the lateral junction, then a small decrease is identified and then increasing till the apex. Thereafter, these patterns found to be steady till $z_{\max}$. It is observed that flow rate is increasing in pure water as compared with copper–water because the density of the copper–water is more and it opposes flow and increases impedance.

The influence of β on shear stress with and without nanoparticles along the inner and outer walls of the daughter artery is depicted in Fig. 8a, b, respectively. From Fig. 8a, it is noticed that shear stress is diminishing with a rise in the value of β and decreasing in the value of ϕ along the inner wall, but

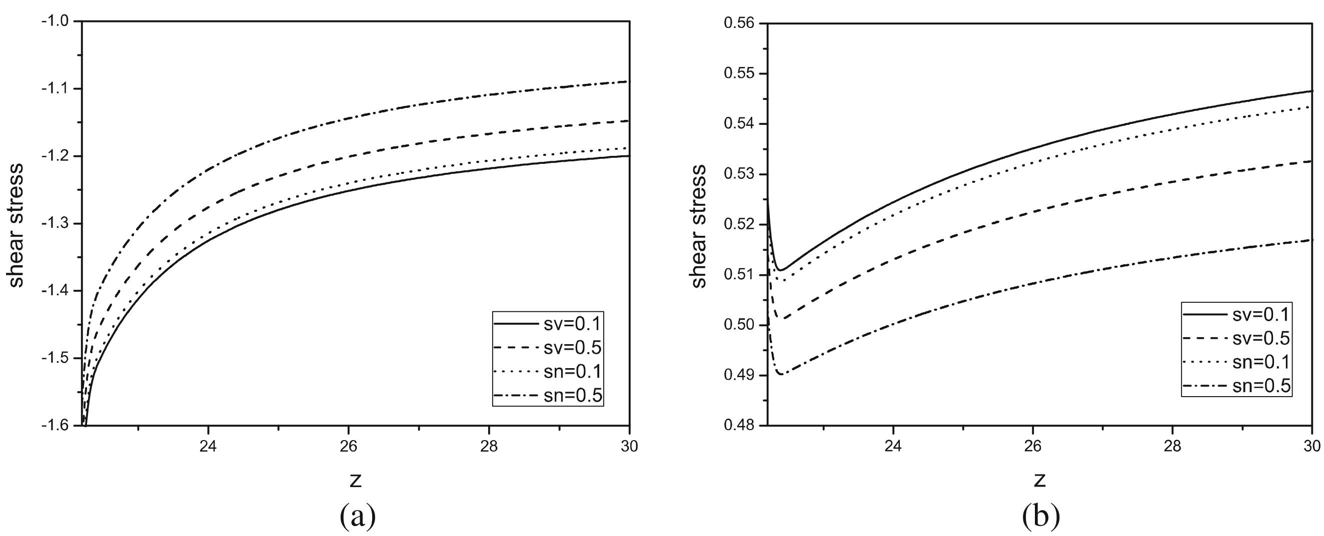

Fig. 10 Influence of s on shear stress along the **a** inner and **b** outer walls of the daughter artery with and without nanoparticles for fixed values of $G_r = 0.3$ and $\beta = \pi/10$