

"5.4.BMP"

CALCULATE THE BOUGUER ADMITTANCE AND COHERENCE DUE TO COMBINED SURFACE AND BURIED LOADING IN CONTINENTAL AND OCEANIC REGIONS.

Mathcad file to calculate the Bouguer admittance and coherence due to combined surface and buried loads in continental and oceanic regions. The method is based on Forsyth (1984) and Ito and Taira (2000) and assumes that the source of the buried loading is located at the base of the crust. The file allows the admittance and coherence to be calculated for any elastic thickness, T_e , ratio of surface and buried loading, f , and crustal thickness.

Application to Continental Regions (see Forsyth, 1984)

Define input parameters

Define elastic thickness, T_e , average gravity, g , Poisson's ratio, ν , Young's Modulus, E , and the Universal Gravitational constant, G . $npts$ is the number of interpolated values. Must be a power of 2.

$$T_e := 80 \text{ km}$$

$$g := 9.81 \text{ m} \cdot \text{sec}^{-2}$$

$$\nu := 0.25$$

$$E := 10^{11} \text{ Pa}$$

$$G := 6.67 \cdot 10^{-11} \text{ newton} \cdot \text{m}^2 \cdot \text{kg}^{-2}$$

$$npts := 1024$$

Define density of the load (i.e. the seafloor topography), water and mantle

$$\rho_{\text{crust}} := 2670 \text{ kg} \cdot \text{m}^{-3}$$

$$\rho_{\text{mantle}} := 3330 \text{ kg} \cdot \text{m}^{-3}$$

Set the ratio of surface to sub-surface loading.

$$f := 1$$

Set the thickness of the crust.

$$\text{continental_thick} := 35 \text{ km}$$

Calculate the flexural rigidity, D (flexure).

$$D := \frac{E \cdot (T_e)^3}{12 \cdot (1 - \nu^2)}$$

Read in an arbitrary-shaped 2-D topographic load

$$M := \text{READPRN}(\text{"seamount.prm"})$$

$$\text{icountt} := \text{rows}(M) - 1$$

$$\text{profile} := (M_{\text{icountt},0} - M_{0,0}) \text{ km}$$

$$j := 0 \dots \text{icountt}$$

$$a_j := M_{j,0} \text{ km}$$

$$b_j := M_{j,1} \text{ m}$$

Interpolate, take mean out and calculate Fourier Transform of the topography

$$dx := \frac{\text{profile}}{npts - 1}$$

$$\text{XKINT} := \frac{2 \cdot \pi}{npts \cdot dx}$$

$$XINT := \frac{\text{profile}}{npts - 1}$$

$$XKINT = 6.277 \times 10^{-6} \cdot m^{-1}$$

$$XKINT := 6.277 \cdot 10^{-7} m^{-1}$$

$$i := 0..npts - 1$$

$$x_i := a_0 + (i \cdot XINT)$$

$$topo_i := \text{linterp}(a, b, x_i)$$

$$\text{mean_depth} := -\text{mean}(topo)$$

$$i := 0..npts - 1$$

$$y_i := topo_i + \text{mean_depth}$$

Calculate the admittance

$$c := \text{fft}(topo)$$

$$k := 0, 1.. \frac{npts}{2}$$

$$\text{wave}_k := XKINT \cdot k$$

$$\text{Phi_dash}_k := \left[\frac{D \cdot (k \cdot XKINT)^4}{g \cdot (\rho_{\text{mantle}} - \rho_{\text{crust}})} + 1 \right]^{-1}$$

(See Eq. 5.3)

$$\text{Phi_3dash}_k := \left[\frac{D \cdot (k \cdot XKINT)^4}{g \cdot (\rho_{\text{mantle}})} + 1 \right]^{-1}$$

(See Eq. 5.37)

Calculate h_i from the observed topography

$$ht_k := \frac{c_k}{\left(1 + \frac{f \cdot \text{Phi_dash}_k \cdot \rho_{\text{crust}}}{\rho_{\text{mantle}} - \rho_{\text{crust}}} \right)}$$

(Substitute Eq. 5.44 for H_b in Eq. 5.50)

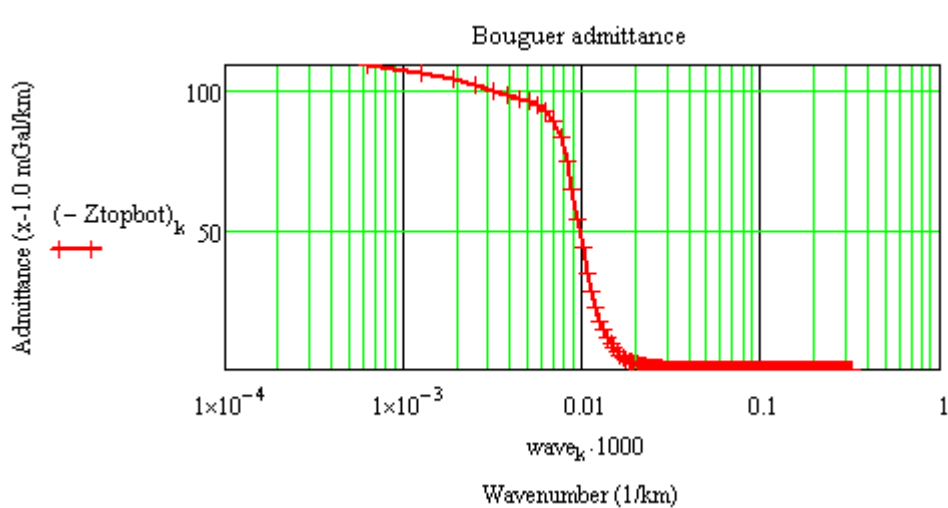
Calculate h_b from h_i

$$hb_k := c_k - ht_k$$

$$\text{factor2}_k := \frac{\left(\frac{hb_k \cdot hb_k}{\text{Phi_3dash}_k} + ht_k \cdot ht_k \cdot \text{Phi_dash}_k \right)}{(hb_k \cdot hb_k + ht_k \cdot ht_k)}$$

$$Z_{\text{topbot}}_k := -2 \cdot \pi \cdot G \cdot 10^5 \cdot e^{-(k \cdot XKINT \cdot \text{mean_depth})} \cdot \rho_{\text{crust}} \cdot 1000 \cdot e^{-k \cdot XKINT \cdot \text{continental_thick}} \cdot \text{factor2}_k$$

(See Eq. 5.43)



Calculate the coherence

$$wt_k := \frac{ht_k \cdot \rho_{crust} \cdot \Phi_{dash_k}}{(\rho_{mantle} - \rho_{crust})}$$

(See Eq. 5.34)

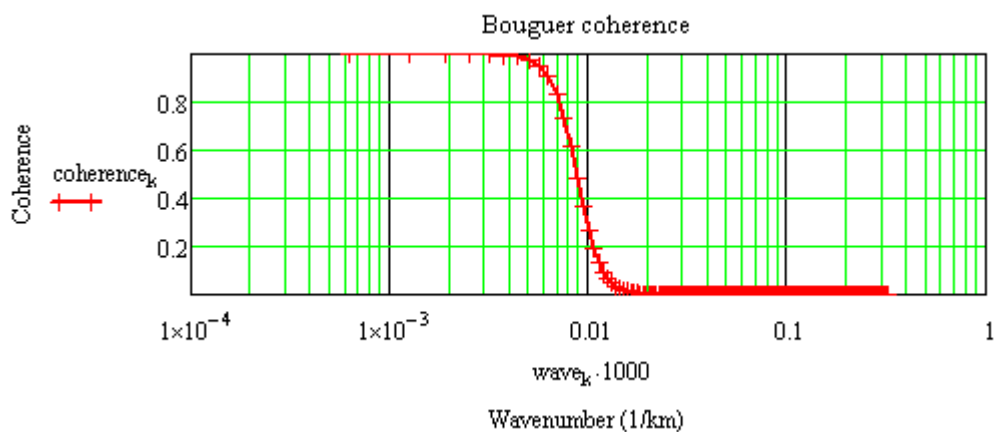
$$wb_k := \frac{hb_k \cdot \rho_{mantle}}{\Phi_{dash_k} \cdot (\rho_{mantle} - \rho_{crust})}$$

(See Eq. 5.36)

(See Forsyth Eq. 25)

$$coherence_k := \frac{\left[\left(ht_k \cdot wt_k + hb_k \cdot wb_k \right) \cdot \left(ht_k \cdot wt_k + hb_k \cdot wb_k \right) \right]}{\left[\left(ht_k \cdot ht_k + hb_k \cdot hb_k \right) \cdot \left(wt_k \cdot wt_k + wb_k \cdot wb_k \right) \right]}$$

(Also Eq. 5.51)



Application to Oceanic Regions (see Ito and Taira, 2000)

Define input parameters

Define elastic thickness, Te , for surface and buried loading.

$Te_{surface} := 4km$

$Te_{buried} := 30km$

Define density of the load (i.e. the seafloor topography), water and mantle.

$\rho_{upper_crust} := 2700kg \cdot m^{-3}$

$\rho_{lower_crust} := 3050kg \cdot m^{-3}$

$\rho_{water} := 1000kg \cdot m^{-3}$

$\rho_{mantle} := 3300kg \cdot m^{-3}$

Set the ratio of surface to sub-surface loading. Note $f=0$ corresponds to surface loading only. The

larger f , the larger is the role of buried loading. Also, the mean water depth and the mean thickness of oceanic crust.

$$f := 2$$

$$\text{mean_depth} := 4.0 \text{ km}$$

$$\text{oceanic_thick} := 21.7 \text{ km}$$

Calculate the flexural rigidity for the surface and sub-surface loads.

$$D_{\text{surface}} := \frac{E \cdot (T_e_{\text{surface}})^3}{12 \cdot (1 - \nu^2)}$$

$$D_{\text{buried}} := \frac{E \cdot (T_e_{\text{buried}})^3}{12 \cdot (1 - \nu^2)}$$

$$XKINT := 6.277 \cdot 10^{-7} \text{ m}^{-1}$$

Calculate the admittance

$$k := 0, 1 \dots 512$$

$$\rho_{\text{nought}} := \rho_{\text{upper_crust}} - \rho_{\text{water}}$$

$$\text{delta_}\rho := \rho_{\text{mantle}} - \rho_{\text{lower_crust}}$$

$$\xi_k := \left[1 + \frac{D_{\text{surface}} \cdot (k \cdot XKINT)^4}{\text{delta_}\rho \cdot g} \right]$$

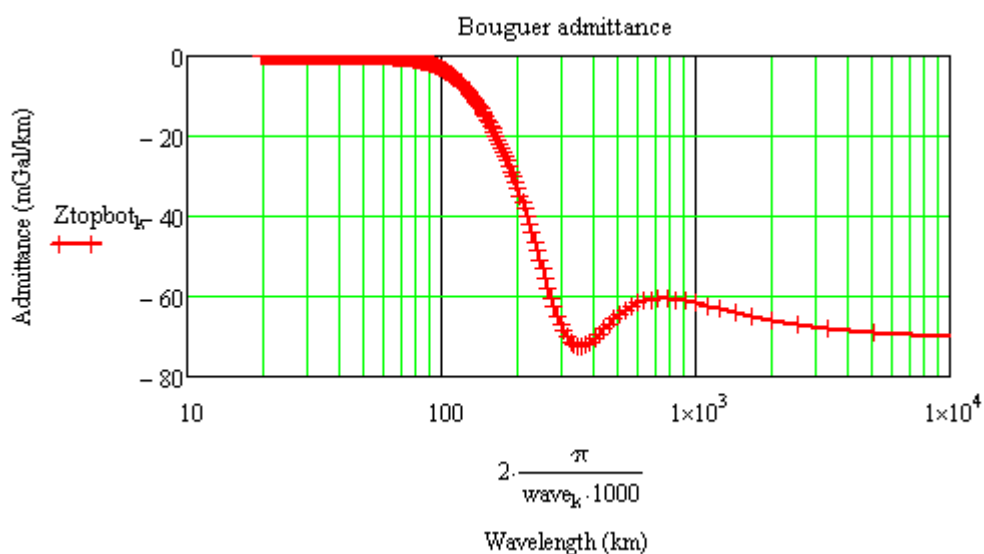
$$\text{wave}_k := XKINT \cdot k$$

$$\phi_k := \left[1 + \frac{D_{\text{buried}} \cdot (k \cdot XKINT)^4}{\rho_{\text{nought}} \cdot g} \right]$$

$$\beta_k := \frac{\left(\frac{\rho_{\text{nought}}}{\text{delta_}\rho} + 1 \right)}{\left(\phi_k \cdot \frac{\rho_{\text{nought}}}{\text{delta_}\rho} + 1 \right)}$$

$$\text{factor}_k := \frac{\left[\left(\frac{1}{\xi_k} \right) + \phi_k \cdot f^2 \cdot (\beta_k)^2 \right]}{\left[1 + f^2 \cdot (\beta_k)^2 \right]}$$

$$Z_{\text{topbot}}_k := -2 \cdot \pi \cdot G \cdot 10^5 \cdot e^{-[k \cdot XKINT \cdot (\text{mean_depth} + \text{oceanic_thick})]} \cdot \rho_{\text{nought}} \cdot 1000 \cdot \text{factor}_k$$



Calculate the coherence

$$\text{coherence}_k := \frac{\left[\left(\frac{1}{\xi_k}\right) + \phi_k \cdot f^2 \cdot (\beta_k)^2\right]^2}{\left[\left(\frac{1}{\xi_k}\right)^2 + (\phi_k)^2 \cdot f^2 \cdot (\beta_k)^2\right] \cdot \left[1 + f^2 \cdot (\beta_k)^2\right]}$$

