Raw data

With ‘1’s and ‘0’s

0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Cormack–Jolly–Seber Model (not a mixture)

Notation: (Pledger et al., 2003)

- K length of study
- ϕ_j probability of survival between time j and $j+1$
- p_j probability of recapture at time j
- f_i first capture time
- ℓ_i last capture time

$$\Pr(\mathbf{CH}_i) = \sum_{d=\ell_i}^{K} \left\{ \left(\prod_{j=f_i}^{d-1} \phi_j \right) (1-\phi_d) \left(\prod_{j=f_i+1}^{d} p_j^{x_{ij}} (1-p_j)^{1-x_{ij}} \right) \right\}$$

Allowing for heterogeneity

A mixture version of $\Pr(\mathbf{CH}_i)$ is given by:

$$\sum_{d=\ell_i}^{K} \sum_{c=1}^{C} \left\{ w_c \left(\prod_{j=f_i}^{d-1} \phi_{jc} \right) (1 - \phi_{dc}) \left(\prod_{j=f_i+1}^{d} p_{jc}^{x_{ij}} (1 - p_{jc})^{1-x_{ij}} \right) \right\}$$

where w_c are the weights from each group C. We define ϕ_{jc} and p_{jc} to be ϕ_j and p_j from group c respectively.

How to model the p_{jc} (or ϕ_{jc}) parameters?

$$\log\left(\frac{p_{jc}}{1-p_{jc}}\right) = \mu_p + \tau_{pj} + \eta_{pc}$$

μ_p overall average

τ_{pj} time component

η_{pc} heterogeneous component

Other link functions are possible, e.g, a linear link function.

We use $\{\phi(\cdot), p(t+h_C)\}$ to denote a model that is constant in ϕ, heterogeneous and time–varying in p and use $\{[\phi(\cdot), p(t+h)]_C\}$ to denote a model that is constant in ϕ, time–varying in p, and heterogeneous in both ϕ and p. (Pledger et al., 2003)

Another example: $\{\phi(t), p(t+h_2)\}$

For $K = 3$, can you spot which parameters are estimable and which aren't?

$$\begin{bmatrix}
1 \\
w_1\left(1-\phi_2+\phi_2\left(1-\mu_p-\tau_{p3}\right)\right)+\left(1-w_1\right)\left(1-\phi_2+\phi_2\left(1-\mu_p-\tau_{p3}-\eta_{p2}\right)\right) \\
w_1\phi_2\left(\mu_p+\tau_{p3}\right)+\left(1-w_1\right)\phi_2\left(\mu_p+\tau_{p3}+\eta_{p2}\right) \\
w_1\left(1-\phi_1+\phi_1\left(1-\mu_p\right)\left(1-\phi_2\right)+\phi_1\left(1-\mu_p\right)\phi_2\left(1-\mu_p-\tau_{p3}\right)\right)+\left(1-w_1\right)\left(1-\phi_1+\phi_1\left(1-\mu_p-\eta_{p2}\right)\left(1-\phi_2\right)+\phi_1\left(1-\mu_p-\eta_{p2}\right)\phi_2\left(1-\mu_p-\tau_{p3}-\eta_{p2}\right)\right) \\
w_1\phi_1\left(1-\mu_p\right)\phi_2\left(\mu_p+\tau_{p3}\right)+\left(1-w_1\right)\phi_1\left(1-\mu_p-\eta_{p2}\right)\phi_2\left(\mu_p+\tau_{p3}+\eta_{p2}\right) \\
w_1\left(\phi_1\mu_p\left(1-\phi_2\right)+\phi_1\mu_p\phi_2\left(1-\mu_p-\tau_{p3}\right)\right)+\left(1-w_1\right)\left(\phi_1\left(\mu_p+\eta_{p2}\right)\left(1-\phi_2\right)+\phi_1\left(\mu_p+\eta_{p2}\right)\phi_2\left(1-\mu_p-\tau_{p3}-\eta_{p2}\right)\right) \\
w_1\phi_1\mu_p\phi_2\left(\mu_p+\tau_{p3}\right)+\left(1-w_1\right)\phi_1\left(\mu_p+\eta_{p2}\right)\phi_2\left(\mu_p+\tau_{p3}+\eta_{p2}\right)
\end{bmatrix}$$

Another example: $\{\phi(t), p(t+h_2)\}$

For $K = 3$, can you spot which parameters are estimable and which aren't?

Expanding brackets

$$\begin{bmatrix} 1 \\ \eta_{p2}\phi_2 w_1 - \eta_{p2}\phi_2 - \mu_p\phi_2 - \phi_2\tau_{p3} + 1 \\ -\eta_{p2}\phi_2 w_1 + \eta_{p2}\phi_2 + \mu_p\phi_2 + \phi_2\tau_{p3} \\ -\eta_{p2}^2\phi_1\phi_2 w_1 - 2\eta_{p2}\mu_p\phi_1\phi_2 w_1 - \eta_{p2}\phi_1\phi_2\tau_{p3} w_1 + \eta_{p2}^2\phi_1\phi_2 + 2\eta_{p2}\mu_p\phi_1\phi_2 + \eta_{p2}\phi_1\phi_2\tau_{p3} + \eta_{p2}\phi_1\phi_2 w_1 + \mu_p^2\phi_1\phi_2 + \mu_p\phi_1\phi_2\tau_{p3} - \eta_{p2}\phi_1\phi_2 + \eta_{p2}\phi_1 w_1 - \mu_p\phi_1\phi_2 - \phi_1\phi_2\tau_{p3} - \eta_{p2}\phi_1 - \mu_p\phi_1 + 1 \\ \eta_{p2}^2\phi_1\phi_2 w_1 + 2\eta_{p2}\mu_p\phi_1\phi_2 w_1 + \eta_{p2}\phi_1\phi_2\tau_{p3} w_1 - \eta_{p2}^2\phi_1\phi_2 - 2\eta_{p2}\mu_p\phi_1\phi_2 - \eta_{p2}\phi_1\phi_2\tau_{p3} - \eta_{p2}\phi_1\phi_2 w_1 - \mu_p^2\phi_1\phi_2 - \mu_p\phi_1\phi_2\tau_{p3} + \eta_{p2}\phi_1\phi_2 + \mu_p\phi_1\phi_2 + \phi_1\phi_2\tau_{p3} \\ \eta_{p2}^2\phi_1\phi_2 w_1 + 2\eta_{p2}\mu_p\phi_1\phi_2 w_1 + \eta_{p2}\phi_1\phi_2\tau_{p3} w_1 - \eta_{p2}^2\phi_1\phi_2 - 2\eta_{p2}\mu_p\phi_1\phi_2 - \eta_{p2}\phi_1\phi_2\tau_{p3} - \mu_p^2\phi_1\phi_2 - \mu_p\phi_1\phi_2\tau_{p3} - \eta_{p2}\phi_1 w_1 + \eta_{p2}\phi_1 + \mu_p\phi_1 \\ -\eta_{p2}^2\phi_1\phi_2 w_1 - 2\eta_{p2}\mu_p\phi_1\phi_2 w_1 - \eta_{p2}\phi_1\phi_2\tau_{p3} w_1 + \eta_{p2}^2\phi_1\phi_2 + 2\eta_{p2}\mu_p\phi_1\phi_2 + \eta_{p2}\phi_1\phi_2\tau_{p3} + \mu_p^2\phi_1\phi_2 + \mu_p\phi_1\phi_2\tau_{p3} \end{bmatrix}$$