$$\begin{cases} \dot{x} = x(1-x) - xy, \\ \dot{y} = y\left(\delta - \dfrac{\beta y}{x}\right) - \dfrac{\alpha y}{\gamma + y}, \end{cases} \tag{3}$$

where $\delta = \frac{r_2}{r_1}, \beta = \frac{r_2}{abK}, \alpha = \frac{aqE}{lr_1^2}$, and $\gamma = \frac{acE}{lr_1}$ are positive constants. Note that after the rescaling transformation, the harvesting term in system (3) is the same as the harvesting term in [33]. From [33], we know that α can be regarded as the maximum harvesting rate of the predator species. When γ equals the number of predator species, harvested biomass reaches one-half of the maximum harvesting rate in system (3). This system characterizes the behavior of a commercial harvesting company when the company's harvesting strategy is determined by both the revenue and cost of harvesting.

In order to obtain the equilibria of system (3), we consider the prey nullcline and predator nullcline of this system, which are given by:

$$\begin{cases} x(1-x) - xy = 0, \\ y\left(\delta - \dfrac{\beta y}{x}\right) - \dfrac{\alpha y}{\gamma + y} = 0. \end{cases} \tag{4}$$

It is obvious that the equilibria are the intersections of these nullclines. We easily see that system (3) possesses a unique boundary equilibrium given by $E_0(1,0)$. For the possible positive equilibria, we only need consider the positive solutions of the following equations:

$$\begin{cases} (\beta+\delta)\,x^2 + (\alpha - \beta\gamma - 2\beta - \gamma\delta - \delta)\,x + \beta\,(\gamma+1) = 0, \\ y = 1 - x. \end{cases} \tag{5}$$

For positive equilibria, x must satisfy $0 < x < 1$. Let Δ denote the discriminant of the first equation of (5) and express Δ in terms of α, i.e.,

$$\Delta(\alpha) = \alpha^2 - 2\,(\beta\gamma + 2\beta + \gamma\delta + \delta)\,\alpha + (\delta + \gamma\delta + \beta\gamma)^2, \tag{6}$$

and let α_1, α_2 be the roots of $\Delta(\alpha)$. An easy calculation shows that

$$\alpha_1 = \beta\gamma + 2\beta + \gamma\delta + \delta - 2\sqrt{\beta(\beta+\delta)(\gamma+1)}, \qquad \alpha_2 = \beta\gamma + 2\beta + \gamma\delta + \delta + 2\sqrt{\beta(\beta+\delta)(\gamma+1)}.$$

Since β, γ and δ are positive parameters, it is easy to check that $\alpha_2 > \alpha_1 > 0$. About the number of equilibria of system (3), we obtain the following theorem.

(c) *If* $\gamma\delta < \alpha < \alpha_1$ *and* $\gamma < \frac{\delta}{\beta}$, *then system* (3) *has two distinct positive equilibria* $E_2(x_2, y_2)$, $E_3(x_3, y_3)$, *where* $x_{2,3} = \frac{\beta\gamma + 2\beta + \gamma\delta + \delta - \alpha \mp \sqrt{\Delta}}{2(\beta+\delta)}$, $y_{2,3} = 1 - x_{2,3}$.

Next we consider the nature of the stability of E_2 and E_3 when $\gamma\delta < \alpha < \alpha_1$ and $\gamma < \frac{\delta}{\beta}$. The Jacobian matrix of system (3) evaluated at the equilibria E_2 and E_3 are given by

$$J_{E_{2,3}} = \begin{pmatrix} 1-2x-y & -x \\ \dfrac{\beta y^2}{x^2} & \delta - \dfrac{2\beta y}{x} - \dfrac{\alpha\gamma}{(\gamma+y)^2} \end{pmatrix}_{(x_{2,3},y_{2,3})}$$
$$= \begin{pmatrix} 1-2x_{2,3}-y_{2,3} & -x_{2,3} \\ \dfrac{\beta y_{2,3}^2}{x_{2,3}^2} & \delta - \dfrac{2\beta y_{2,3}}{x_{2,3}} - \dfrac{\alpha\gamma}{(\gamma+y_{2,3})^2} \end{pmatrix}, \tag{23}$$

and the determinant and the trace of the Jacobian matrix are given by

$$\begin{aligned} \mathrm{Det}\left[J_{E_{2,3}}\right] &= x_{2,3}\left[\frac{2\beta y_{2,3}}{x_{2,3}} + \frac{\alpha\gamma}{(\gamma+y_{2,3})^2} + \frac{\beta y_{2,3}^2}{x_{2,3}^2} - \delta\right], \\ \mathrm{Tr}\left[J_{E_{2,3}}\right] &= -x_{2,3} + \delta - \frac{2\beta y_{2,3}}{x_{2.3}} - \frac{\alpha\gamma}{(\gamma+y_{2.3})^2}. \end{aligned} \tag{24}$$

4.3. Hopf bifurcation

In the previous section we have shown that E_3 is always a saddle whenever it exists and presented the conditions required for local asymptotic stability of E_2. Furthermore, it can be easily concluded that the equilibrium E_2 may lose its stability through Hopf bifurcation under certain parametric restrictions. Considering α as the bifurcation parameter, the Hopf bifurcation threshold is a positive root of $\mathrm{Tr}\,[J_{E_2}] = 0$, say $\alpha = \alpha_H$ which satisfy $\mathrm{Det}\,[J_{E_2}]\big|_{\alpha=\alpha_H} > 0$. The stability property of E_2 changes when α passes through the critical magnitude $\alpha = \alpha_H$. Thus we summarize our findings in the following theorem.

Theorem 9. *Assume that system parameters satisfy the conditions for existence of interior equilibrium given in Theorem 1(c), then the interior equilibrium E_2 changes its stability through Hopf bifurcation threshold $\alpha = \alpha_H$.*

Proof. In order to ensure the changes of stability through non-degenerate Hopf bifurcation, we need to verify the transversality condition for Hopf bifurcation. Obviously,

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\mathrm{Tr}\,[J_{E_2}]\bigg|_{\alpha=\alpha_H} = \frac{\gamma\left[(\beta+\delta)x_2 - \beta\right]^2}{\alpha^2 x_2^2}\bigg|_{\alpha=\alpha_H} \neq 0.$$

The interior equilibrium E_2 loses its stability through non-degenerate Hopf bifurcation when the parametric restriction $\mathrm{Tr}\,[J_{E_2}] = 0$ and the transversality condition mentioned above are satisfied simultaneously.