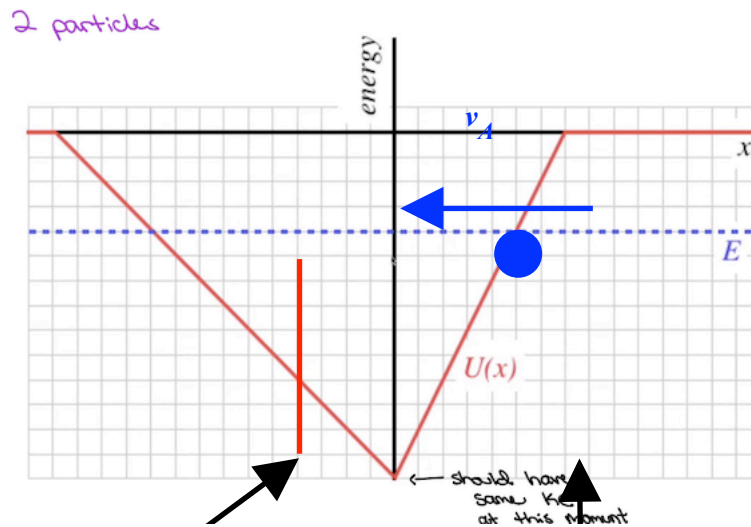


Problem 2: *At the moment they come to rest, where is the center of mass of the 2 particle system?*



Looking at the figure above, we can see in the energy diagram where the kinetic energy for each particle goes to zero. For Particle A, that is at $x = -10$, and for Particle B, that is at $x = 5$. I am assuming that each square has a width of 1-unit. In general the center of mass for a 2 particle system at any given time will be:

$$x_{cm} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$$

So substituting in what we know so far:

$$x_{cm} = \frac{-10 m_A + 5 m_B}{m_A + m_B}$$

But we can find out more. If our particle's come to rest at the same time, that means that Particle A has to travel a distance $\Delta x = 10$ in the same distance that Particle B travels a distance $\Delta x = 5$. Now, we know that:

$$-\frac{\partial}{\partial x} U(x) = F$$

In other words, the negative of the slope of the potential is the force. In this case, that means the force applied to each particle on either side of the origin is constant. Therefore the magnitude of the force to the left of $x = 0$ is 1, while the magnitude of the force to the right of the origin is 2. Let's use those forces to find the speed of each particle at $x = 0$. Particle A is originally at rest at $x = 5$, while Particle B is originally at rest at $x = -10$. Their velocities at the origin will then be:

$$v_A = \sqrt{2 a_A \Delta x_A} = \sqrt{2 \left(\frac{2}{m_A} \right) 5} = 2 \sqrt{\frac{5}{m_A}}$$

$$v_B = \sqrt{2 a_B \Delta x_B} = \sqrt{2 \left(\frac{1}{m_B} \right) 10} = 2 \sqrt{\frac{5}{m_B}}$$

Since both particles come to a rest at the same time and pass each other at the origin at the same time, and since $t = \frac{\Delta x}{v}$, we get:

$$\frac{\Delta x_A}{v_A} = \frac{\Delta x_B}{v_B}$$

$$\frac{5}{2 \sqrt{\frac{5}{m_A}}} = \frac{10}{2 \sqrt{\frac{5}{m_B}}}$$

$$\frac{\sqrt{\frac{5}{m_B}}}{\sqrt{\frac{5}{m_A}}} = \left(\frac{10}{2} \right) \cdot \left(\frac{2}{5} \right)$$

$$\sqrt{\frac{m_A}{m_B}} = 2$$

$$\frac{m_A}{m_B} = 4$$

$$m_A = 4 m_B$$

Now we can complete our center of mass calculation.

From above:

$$x_{cm} = \frac{-10 m_A + 5 m_B}{m_A + m_B}$$

$$x_{cm} = \frac{-10 \cdot (4 m_B) + 5 m_B}{4 m_B + m_B}$$

$$x_{cm} = -\frac{35 m_B}{5 m_B} = -7$$

ANSWER $\rightarrow x_{cm} = -7$

Note: I left values for forces, energy, position, and velocity unitless in my calculations because they weren't specified in the problem.