$$A\left(\frac{\partial u}{\partial t}-S\frac{\partial u}{\partial y}\right)=D\frac{\partial^2 u}{\partial y^2}+BG_1\theta-\left(M_1+\frac{D}{K}\right)u, \tag{12}$$

$$C\left(\frac{\partial \theta}{\partial t}-S\frac{\partial \theta}{\partial y}-Q_L\varphi\right)=\frac{1}{P_r}\left(E\frac{\partial^2 \theta}{\partial y^2}-F\theta+D_u\frac{\partial^2 \varphi}{\partial y^2}\right), \tag{13}$$

$$\frac{\partial \varphi}{\partial t}-S\frac{\partial \varphi}{\partial y}=\frac{1}{S_c}\frac{\partial^2 \varphi}{\partial y^2}-K_r\varphi, \tag{14}$$

with $M_1 = m, F = Q + R$. Along with the boundary circumstances

$$\left.\begin{array}{l} t<0, u=0, \theta=0, \varphi=0, \\ t\geq 0, u=1, \theta=1+\varepsilon e^{i\omega t}, \varphi=1+\varepsilon e^{i\omega t} as, y=0, \\ u=0, \theta=0, \varphi=0 as, y\to\infty. \end{array}\right\} \tag{15}$$

3. Solution methodology

The coupled non-linear partial differential equation (12) through (14) can be solved in closed form. The perturbation technique is used to present the analytical simulations. The motivations for using the perturbation method are due to higher accuracy. The simulations for this analytical procedure are proceeded as:

$$u(y,t)=u_0+\varepsilon u_1 e^{i\omega t} \tag{16}$$

$$\theta(y,t)=\theta_0+\varepsilon \theta_1 e^{i\omega t} \tag{17}$$

$$\varphi(y,t)=\varphi_0+\varepsilon \varphi_1 e^{i\omega t} \tag{18}$$

where $\varepsilon \ll 1$ is a parameter

We obtain the following set of ordinary differential equations by substituting (16) to (18) in equations (12)–(14) and by equating zeroth and first order equations.

$$Du_0''+ASu_0'-\left(M_1+\frac{D}{K}\right)u_0=-BG_1\theta_0 \tag{19}$$

$$Du_1''+ASu_1'-\left(\left(M_1+\frac{D}{K}\right)+Ai\omega\right)u_1=-BG_1\theta_1 \tag{20}$$

Here$G_1 = G_r \cos\gamma$

$$E\theta_0''+P_rCS\theta_0'-\mathrm{F}\,\theta_0=-\mathrm{D_u}\varphi_0''-\mathrm{P_r C\,Q_L}\varphi_0 \tag{21}$$

$$E\theta_1''+P_rCS\theta_1'-(\mathrm{FP_r Ci}\omega)\,\theta_1=-\mathrm{D_u}\varphi_1''-\mathrm{P_r C\,Q_L}\varphi_1 \tag{22}$$

$$\varphi_0''+SS_c\varphi_0'-K_rS_c\varphi_0=0 \tag{23}$$

$$\varphi_1''+SS_c\varphi_1'-(i\omega+K_r)S_c\varphi_1=0 \tag{24}$$

The result of the boundary conditions equation (15) is

$$\left.\begin{array}{l} u_0=1; u_1=0; \theta_0=1, \theta_1=1, \varphi_0=1, \varphi_1=1, at; y=0, \\ u_0=0; u_1=0; \theta_0=0, \theta_1=0, \varphi_0=0, \varphi_1=0, as, y\to\infty. \end{array}\right\} \tag{25}$$

By substituting the boundary conditions (25) in equations (19)–(24) we get

$$\begin{aligned} u_0&=B_5e^{-m_5y}+B_3e^{-m_3y}+B_4e^{-m_1y} \\ u_1&=B_8e^{-m_6y}+B_6e^{-m_4y}+B_7e^{-m_2y} \\ \theta_0&=B_1e^{-m_3y}+A_1e^{-m_1y} \\ \theta_1&=B_2e^{-m_4y}+A_2e^{-m_2y} \\ \varphi_0&=e^{-m_1y} \\ \varphi_1&=e^{-m_2y} \end{aligned} \tag{26}$$